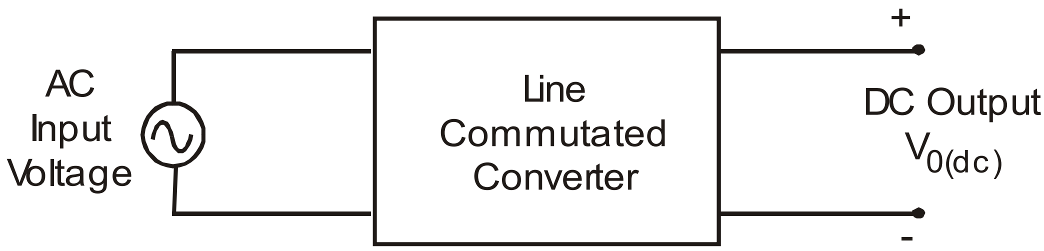
CONTROLLED RECTIFIERS
(Line Commutated AC to DC converters)

INTRODUCTION TO CONTROLLED RECTIFIERS

Controlled rectifiers are line commutated ac to dc power converters which are used to convert a fixed voltage, fixed frequency ac power supply into variable dc output voltage.



Type of input: Fixed voltage, fixed frequency ac power supply.
Type of output: Variable dc output voltage

The input supply fed to a controlled rectifier is ac supply at a fixed rms voltage and at a fixed frequency. We can obtain variable dc output voltage by using controlled rectifiers. By employing phase controlled thyristors in the controlled rectifier circuits we can obtain variable dc output voltage and variable dc (average) output current by varying the trigger angle (phase angle) at which the thyristors are triggered. We obtain a uni-directional and pulsating load current waveform, which has a specific average value.

The thyristors are forward biased during the positive half cycle of input supply and can be turned ON by applying suitable gate trigger pulses at the thyristor gate leads. The thyristor current and the load current begin to flow once the thyristors are triggered (turned ON) say at $\omega t = \alpha$. The load current flows when the thyristors conduct from $\omega t = \alpha$ to β. The output voltage across the load follows the input supply voltage through the conducting thyristor. At $\omega t = \beta$, when the load current falls to zero, the thyristors turn off due to AC line (natural) commutation.

In some bridge controlled rectifier circuits the conducting thyristor turns off, when the other thyristor is (other group of thyristors are) turned ON.

The thyristor remains reverse biased during the negative half cycle of input supply. The type of commutation used in controlled rectifier circuits is referred to AC line commutation or Natural commutation or AC phase commutation.

When the input ac supply voltage reverses and becomes negative during the negative half cycle, the thyristor becomes reverse biased and hence turns off. There are several types of power converters which use ac line commutation. These are referred to as line commutated converters.

Different types of line commutated converters are

- Phase controlled rectifiers which are AC to DC converters.
- AC to AC converters

- AC voltage controllers, which convert input ac voltage into variable ac output voltage at the same frequency.
- Cyclo converters, which give low output frequencies.

All these power converters operate from ac power supply at a fixed rms input supply voltage and at a fixed input supply frequency. Hence they use ac line commutation for turning off the thyristors after they have been triggered ON by the gating signals.

DIFFERENCES BETWEEN DIODE RECTIFIERS AND PHASE CONTROLLED RECTIFIERS

The diode rectifiers are referred to as uncontrolled rectifiers which make use of power semiconductor diodes to carry the load current. The diode rectifiers give a fixed dc output voltage (fixed average output voltage) and each diode rectifying element conducts for one half cycle duration (T/2 seconds), that is the diode conduction angle = 180^0 or π radians.

A single phase half wave diode rectifier gives (under ideal conditions) an average dc output voltage $V_{O(dc)} = \frac{V_m}{\pi}$ and single phase full wave diode rectifier gives (under ideal conditions) an average dc output voltage $V_{O(dc)} = \frac{2V_m}{\pi}$, where V_m is maximum value of the available ac supply voltage.

Thus we note that we can not control (we can not vary) the dc output voltage or the average dc load current in a diode rectifier circuit.

In a phase controlled rectifier circuit we use a high current and a high power thyristor device (silicon controlled rectifier; SCR) for conversion of ac input power into dc output power.

Phase controlled rectifier circuits are used to provide a variable voltage output dc and a variable dc (average) load current.

We can control (we can vary) the average value (dc value) of the output load voltage (and hence the average dc load current) by varying the thyristor trigger angle.

We can control the thyristor conduction angle δ from 180^0 to 0^0 by varying the trigger angle α from 0^0 to 180^0, where thyristor conduction angle $\delta = (\pi - \alpha)$

APPLICATIONS OF PHASE CONTROLLED RECTIFIERS

- DC motor control in steel mills, paper and textile mills employing dc motor drives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.
- Reactor controls.
- Portable hand tool drives.
- Variable speed industrial drives.
- Battery charges.
- High voltage DC transmission.
- Uninterruptible power supply systems (UPS).

Some years back ac to dc power conversion was achieved using motor generator sets, mercury arc rectifiers, and thyratorn tubes. The modern ac to dc power converters are designed using high power, high current thyristors and presently most of the ac-dc power converters are thyristorised power converters. The thyristor devices are phase controlled to obtain a variable dc output voltage across the output load terminals. The phase controlled thyristor converter uses ac line commutation (natural commutation) for commutating (turning off) the thyristors that have been turned ON.

The phase controlled converters are simple and less expensive and are widely used in industrial applications for industrial dc drives. These converters are classified as two quadrant converters if the output voltage can be made either positive or negative for a given polarity of output load current. There are also single quadrant ac-dc converters where the output voltage is only positive and cannot be made negative for a given polarity of output current. Of course single quadrant converters can also be designed to provide only negative dc output voltage.

The two quadrant converter operation can be achieved by using fully controlled bridge converter circuit and for single quadrant operation we use a half controlled bridge converter.

CLASSIFICATION OF PHASE CONTROLLED RECTIFIERS

The phase controlled rectifiers can be classified based on the type of input power supply as

- Single Phase Controlled Rectifiers which operate from single phase ac input power supply.
- Three Phase Controlled Rectifiers which operate from three phase ac input power supply.

DIFFERENT TYPES OF SINGLE PHASE CONTROLLED RECTIFIERS

*Single Phase Controlled Rectifier*s are further subdivided into different types

- *Half wave controlled rectifier* which uses a single thyristor device (which provides output control only in one half cycle of input ac supply, and it provides low dc output).

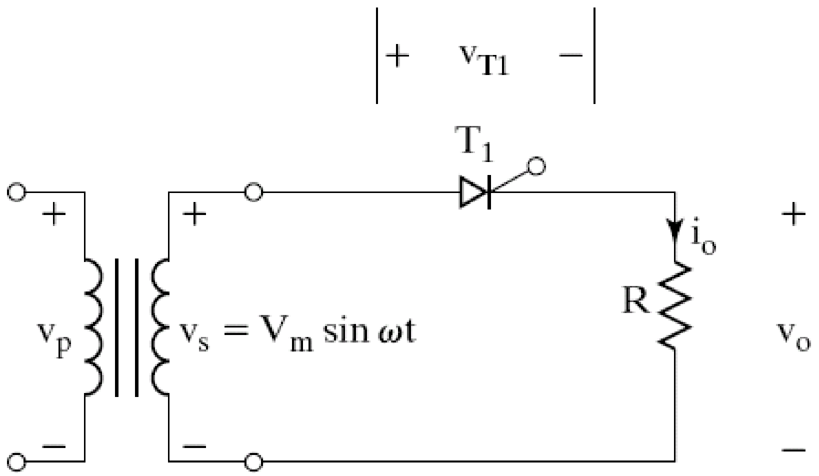
- *Full wave controlled rectifiers* (which provide higher dc output)
    - Full wave controlled rectifier using a center tapped transformer (which requires two thyristors).
    - Full wave bridge controlled rectifiers (which do not require a center tapped transformer)
        - *Single phase semi-converter* (half controlled bridge converter, using two SCR's and two diodes, to provide single quadrant operation).
        - *Single phase full converter* (fully controlled bridge converter which requires four SCR's, to provide two quadrant operation).

*Three Phase Controlled Rectifier*s are of different types

- Three phase half wave controlled rectifiers.
- Three phase full wave controlled rectiriers.
    - Semi converter (half controlled bridge converter).
    - Full converter (fully controlled bridge converter).

PRINCIPLE OF PHASE CONTROLLED RECTIFIER OPERATION

The basic principle of operation of a phase controlled rectifier circuit is explained with reference to a single phase half wave phase controlled rectifier circuit with a resistive load shown in the figure.



$R = R_L =$ Load Resistance

Fig.: Single Phase Half-Wave Thyristor Converter with a Resistive Load

A single phase half wave thyristor converter which is used for ac-dc power conversion is shown in the above figure. The input ac supply is obtained from a main supply transformer to provide the desired ac supply voltage to the thyristor converter depending on the output dc voltage required. v_P represents the primary input ac supply voltage. v_S represents the secondary ac supply voltage which is the output of the transformer secondary.

During the positive half cycle of input supply when the upper end of the transformer secondary is at a positive potential with respect to the lower end, the thyristor anode is positive with respect to its cathode and the thyristor is in a forward biased state. The thyristor is triggered at a delay angle of $\omega t = \alpha$, by applying a suitable gate trigger pulse to the gate lead of thyristor. When the thyristor is triggered at a delay angle of $\omega t = \alpha$, the thyristor conducts and assuming an ideal thyristor, the thyristor behaves as a closed switch and the input supply voltage appears across the load when the thyristor conducts from $\omega t = \alpha$ to π radians. Output voltage $v_O = v_S$, when the thyristor conducts from $\omega t = \alpha$ to π.

For a purely resistive load, the load current i_O (output current) that flows when the thyristor T_1 is on, is given by the expression

$$i_O = \frac{v_O}{R_L}, \text{ for } \alpha \le \omega t \le \pi$$

The output load current waveform is similar to the output load voltage waveform during the thyristor conduction time from α to π. The output current and the output voltage waveform are in phase for a resistive load. The load current increases as the input supply voltage increases and the maximum load current flows at $\omega t = \frac{\pi}{2}$, when the input supply voltage is at its maximum value.

The maximum value (peak value) of the load current is calculated as

$$i_{O(\max)} = I_m = \frac{V_m}{R_L}.$$

Note that when the thyristor conducts (T_1 is on) during $\omega t = \alpha$ to π, the thyristor current i_{T1}, the load current i_O through R_L and the source current i_S flowing through the transformer secondary winding are all one and the same.

Hence we can write

$$i_S = i_{T1} = i_O = \frac{v_O}{R} = \frac{V_m \sin \omega t}{R} \; ; \text{ for } \alpha \le \omega t \le \pi$$

I_m is the maximum (peak) value of the load current that flows through the transformer secondary winding, through T_1 and through the load resistor R_L at the instant $\omega t = \frac{\pi}{2}$, when the input supply voltage reaches its maximum value.

When the input supply voltage decreases the load current decreases. When the supply voltage falls to zero at $\omega t = \pi$, the thyristor and the load current also falls to zero at $\omega t = \pi$. Thus the thyristor naturally turns off when the current flowing through it falls to zero at $\omega t = \pi$.

During the negative half cycle of input supply when the supply voltage reverses and becomes negative during $\omega t = \pi$ to 2π radians, the anode of thyristor is at a negative potential with respect to its cathode and as a result the thyristor is reverse biased and hence it remains cut-off (in the reverse blocking mode). The thyristor cannot conduct during its reverse biased state between $\omega t = \pi$ to 2π. An ideal thyristor under reverse biased condition behaves as an open switch and hence the load current and load voltage are zero during $\omega t = \pi$ to 2π. The maximum or peak reverse voltage that appears across the thyristor anode and cathode terminals is V_m.

The trigger angle α (delay angle or the phase angle α) is measured from the beginning of each positive half cycle to the time instant when the gate trigger pulse is applied. The thyristor conduction angle is from α to π, hence the conduction angle $\delta = (\pi - \alpha)$. The maximum conduction angle is π radians (180^0) when the trigger angle $\alpha = 0$.

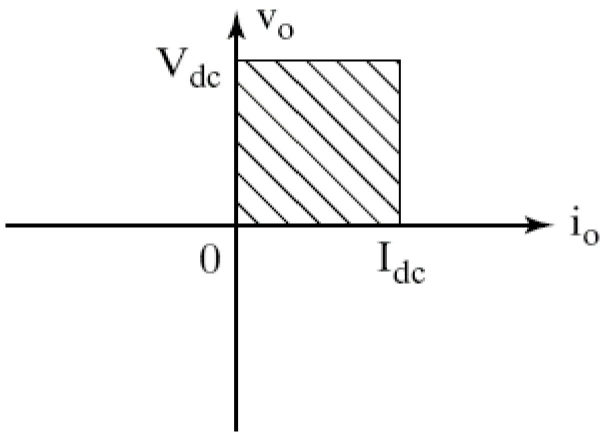


Fig: Quadrant Diagram

The waveforms shows the input ac supply voltage across the secondary winding of the transformer which is represented as v_S, the output voltage across the load, the output (load) current, and the thyristor voltage waveform that appears across the anode and cathode terminals.

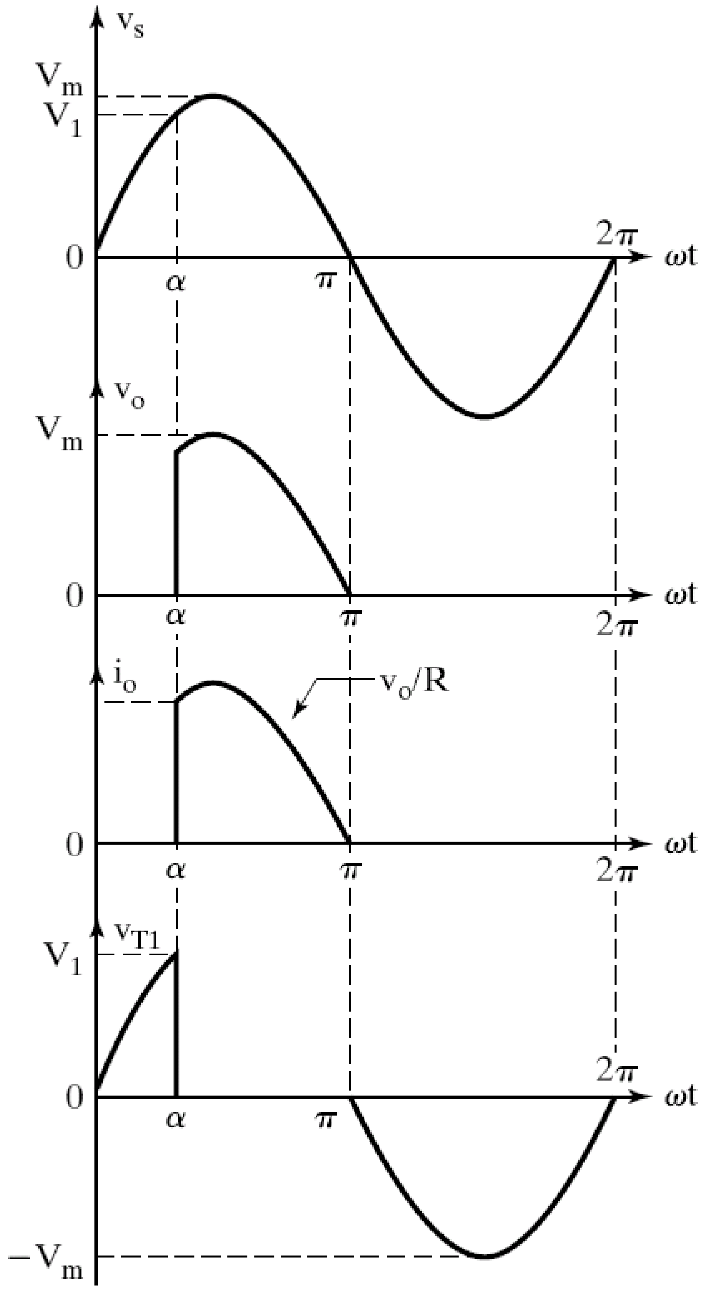


Fig: Waveforms of single phase half-wave controlled rectifier with resistive load

EQUATIONS

$v_s = V_m \sin \omega t =$ the ac supply voltage across the transformer secondary.

$V_m =$ max. (peak) value of input ac supply voltage across transformer secondary.

$V_S = \frac{V_m}{\sqrt{2}} =$ RMS value of input ac supply voltage across transformer secondary.

$v_O = v_L =$ the output voltage across the load ; $i_O = i_L =$ output (load) current.

When the thyristor is triggered at $\omega t = \alpha$ (an ideal thyristor behaves as a closed switch) and hence the output voltage follows the input supply voltage.

$v_O = v_L = V_m \sin \omega t$; for $\omega t = \alpha$ to π , when the thyristor is on.

$i_O = i_L = \frac{v_O}{R}$ = Load current for $\omega t = \alpha$ to π , when the thyristor is on.

TO DERIVE AN EXPRESSION FOR THE AVERAGE (DC) OUTPUT VOLTAGE ACROSS THE LOAD

If V_m is the peak input supply voltage, the average output voltage V_{dc} can be found from

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\int_{\alpha}^{\pi} \sin \omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos \omega t \Big/_{\alpha}^{\pi}\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos \pi + \cos \alpha\right] \qquad ; \ \cos \pi = -1$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1 + \cos \alpha\right] \quad ; \quad V_m = \sqrt{2} V_S$$

The maximum average (dc) output voltage is obtained when $\alpha = 0$ and the maximum dc output voltage $V_{dc(\max)} = V_{dm} = \frac{V_m}{\pi}$.

The average dc output voltage can be varied by varying the trigger angle α from 0 to a maximum of $180^0 \left(\pi \text{ radians}\right)$.

We can plot the control characteristic, which is a plot of dc output voltage versus the trigger angle α by using the equation for $V_{O(dc)}$.

CONTROL CHARACTERISTIC OF SINGLE PHASE HALF WAVE PHASE CONTROLLED RECTIFIER WITH RESISTIVE LOAD

The average dc output voltage is given by the expression

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right]$$

We can obtain the control characteristic by plotting the expression for the dc output voltage as a function of trigger angle α

Trigger angle α in degrees	$V_{O(dc)}$	**%**
0	$V_{dm} = \frac{V_m}{\pi}$	100% V_{dm}
30^0	0.933 V_{dm}	93.3 % V_{dm}
60^0	0.75 V_{dm}	75 % V_{dm}
90^0	0.5 V_{dm}	50 % V_{dm}
120^0	0.25 V_{dm}	25 % V_{dm}
150^0	0.06698 V_{dm}	6.69 % V_{dm}
180^0	0	0

$$V_{dm} = \frac{V_m}{\pi} = V_{dc(\max)}$$

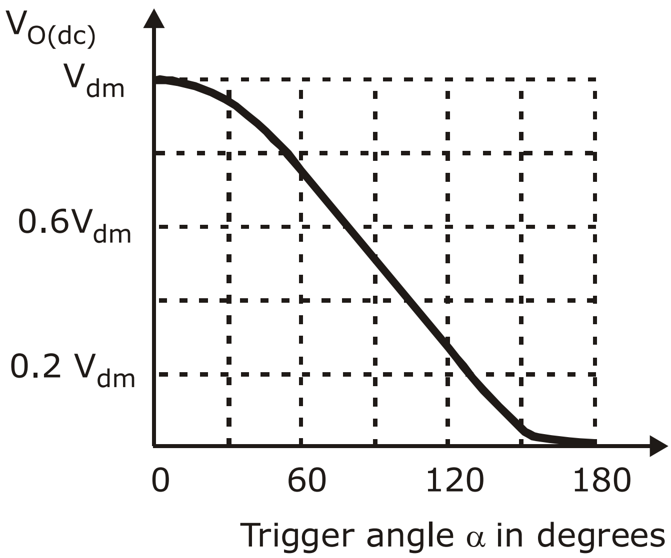


Fig.: Control characteristic

Normalizing the dc output voltage with respect to V_{dm}, the normalized output voltage

$$V_{dcn} = \frac{V_{O(dc)}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \frac{\frac{V_m}{2\pi}\left(1+\cos\alpha\right)}{\frac{V_m}{\pi}}$$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{2}\left(1+\cos\alpha\right) = V_{dcn}$$

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT VOLTAGE OF A SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RESISTIVE LOAD

The rms output voltage is given by

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_0^{2\pi} v_O^2 .d\left(\omega t\right)\right]$$

Output voltage $v_O = V_m \sin\omega t$; for $\omega t = \alpha$ to π

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_\alpha^{\pi} V_m^2 \sin^2 \omega t .d\left(\omega t\right)\right]^{\frac{1}{2}}$$

By substituting $\sin^2\omega t = \frac{1-\cos 2\omega t}{2}$, we get

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_\alpha^{\pi} V_m^2 \frac{\left(1-\cos 2\omega t\right)}{2} .d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi}\int_\alpha^{\pi}\left(1-\cos 2\omega t\right) d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi}\left\{\int_\alpha^{\pi} d\left(\omega t\right) - \int_\alpha^{\pi}\cos 2\omega t\, d\left(\omega t\right)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left\{\left(\omega t\right)\Big/_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\left(\pi-\alpha\right) - \frac{\left(\sin 2\pi - \sin 2\alpha\right)}{2}\right)\right]^{\frac{1}{2}} \; ; \; \sin 2\pi = 0$$

Hence we get,

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left((\pi-\alpha)+\frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\left((\pi-\alpha)+\frac{\sin 2\alpha}{2}\right)^{\frac{1}{2}}$$

PERFORMANCE PARAMETERS OF PHASE CONTROLLED RECTIFIERS

Output dc power (average or dc output power delivered to the load)

$$P_{O(dc)} = V_{O(dc)} \times I_{O(dc)} \text{ ; i.e., } P_{dc} = V_{dc} \times I_{dc}$$

Where

$V_{O(dc)} = V_{dc} =$ average or dc value of output (load) voltage.

$I_{O(dc)} = I_{dc} =$ average or dc value of output (load) current.

Output ac power

$$P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}$$

Efficiency of Rectification (Rectification Ratio)

$$\text{Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \text{ ; } \qquad \% \text{ Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \times 100$$

The output voltage can be considered as being composed of two components

- The dc component $V_{O(dc)}$ = DC or average value of output voltage.
- The ac component or the ripple component $V_{ac} = V_{r(rms)} =$ RMS value of all the ac ripple components.

The total RMS value of output voltage is given by

$$V_{O(RMS)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}$$

Therefore

$$V_{ac} = V_{r(rms)} = \sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}$$

Form Factor (FF) which is a measure of the shape of the output voltage is given by

$$FF = \frac{V_{O(RMS)}}{V_{O(dc)}} = \frac{\text{RMS output (load) voltage}}{\text{DC output (load) voltage}}$$

The Ripple Factor (RF) which is a measure of the ac ripple content in the output voltage waveform. The output voltage ripple factor defined for the output voltage waveform is given by

$$r_v = RF = \frac{V_{r(rms)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}$$

$$r_v = \frac{\sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}}{V_{O(dc)}} = \sqrt{\left[\frac{V_{O(RMS)}}{V_{O(dc)}}\right]^2 - 1}$$

Therefore

$$r_v = \sqrt{FF^2 - 1}$$

Current Ripple Factor defined for the output (load) current waveform is given by

$$r_i = \frac{I_{r(rms)}}{I_{O(dc)}} = \frac{I_{ac}}{I_{dc}}$$

Where $$I_{r(rms)} = I_{ac} = \sqrt{I_{O(RMS)}^2 - I_{O(dc)}^2}$$

Some times the peak to peak output ripple voltage is also considered to express the peak to peak output ripple voltage as

$$V_{r(pp)} = \text{ peak to peak ac ripple output voltage}$$

The peak to peak ac ripple load current is the difference between the maximum and the minimum values of the output load current.

$$I_{r(pp)} = I_{O(\max)} - I_{O(\min)}$$

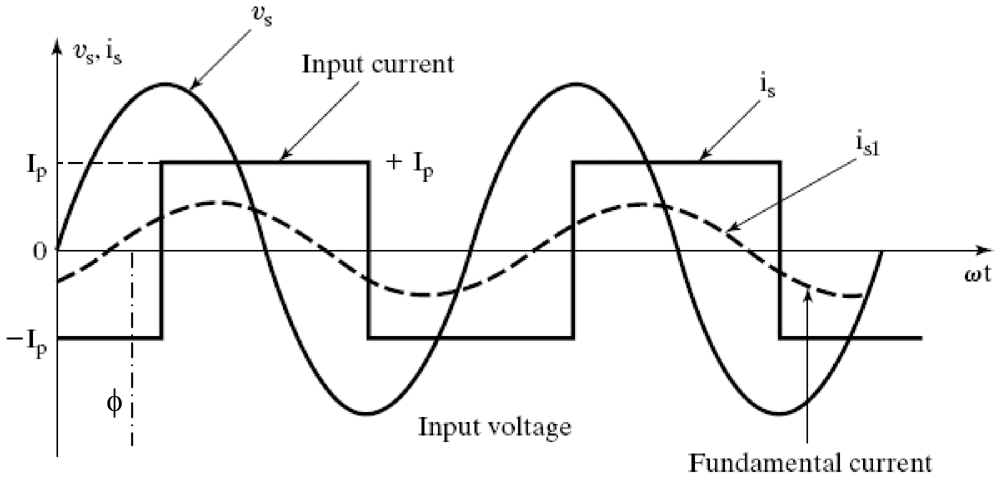
Transformer Utilization Factor (TUF)

$$TUF = \frac{P_{O(dc)}}{V_S \times I_S}$$

Where

$V_S =$ RMS value of transformer secondary output voltage (RMS supply voltage at the secondary)

$I_S =$ RMS value of transformer secondary current (RMS line or supply current).



$v_S =$ Supply voltage at the transformer secondary side.

$i_S =$ Input supply current (transformer secondary winding current).

$i_{S1} =$ Fundamental component of the input supply current.

$I_P =$ Peak value of the input supply current.

$\phi =$ Phase angle difference between (sine wave components) the fundamental components of input supply current and the input supply voltage.

$\phi =$ Displacement angle (phase angle)

For an RL load $\phi =$ Displacement angle = Load impedance angle

$$\therefore \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ for an RL load}$$

Displacement Factor (DF) *or* ***Fundamental Power Factor***

$$DF = Cos\phi$$

Harmonic Factor (HF) or Total Harmonic Distortion Factor (THD)

The harmonic factor is a measure of the distortion in the output waveform and is also referred to as the total harmonic distortion (THD)

$$HF = \left[\frac{I_S^2 - I_{S1}^2}{I_{S1}^2}\right]^{\frac{1}{2}} = \left[\left(\frac{I_S}{I_{S1}}\right)^2 - 1\right]^{\frac{1}{2}}$$

Where

$I_S =$ RMS value of input supply current.

$I_{S1} =$ RMS value of fundamental component of the input supply current.

Input Power Factor (PF)

$$PF = \frac{V_S I_{S1}}{V_S I_S}\cos\phi = \frac{I_{S1}}{I_S}\cos\phi$$

The Crest Factor (CF)

$$CF = \frac{I_{S(peak)}}{I_S} = \frac{\text{Peak input supply current}}{\text{RMS input supply current}}$$

For an Ideal Controlled Rectifier

$FF = 1$; which means that $V_{O(RMS)} = V_{O(dc)}$.

Efficiency $\eta = 100\%$; which means that $P_{O(dc)} = P_{O(ac)}$.

$V_{ac} = V_{r(rms)} = 0$; so that $RF = r_v = 0$; Ripple factor = 0 (ripple free converter).

$TUF = 1$; which means that $P_{O(dc)} = V_S \times I_S$

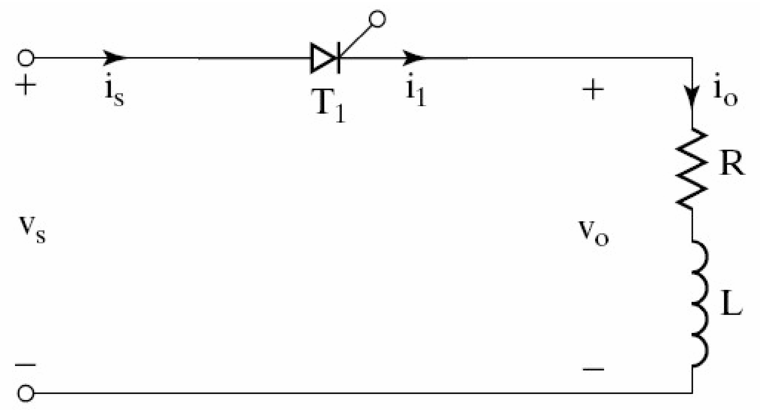
$HF = THD = 0$; which means that $I_S = I_{S1}$

$PF = DPF = 1$; which means that $\phi = 0$

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH AN RL LOAD

In this section we will discuss the operation and performance of a single phase half wave controlled rectifier with RL load. In practice most of the loads are of RL type. For example if we consider a single phase controlled rectifier controlling the speed of a dc motor, the load which is the dc motor winding is an RL type of load, where R represents the motor winding resistance and L represents the motor winding inductance.

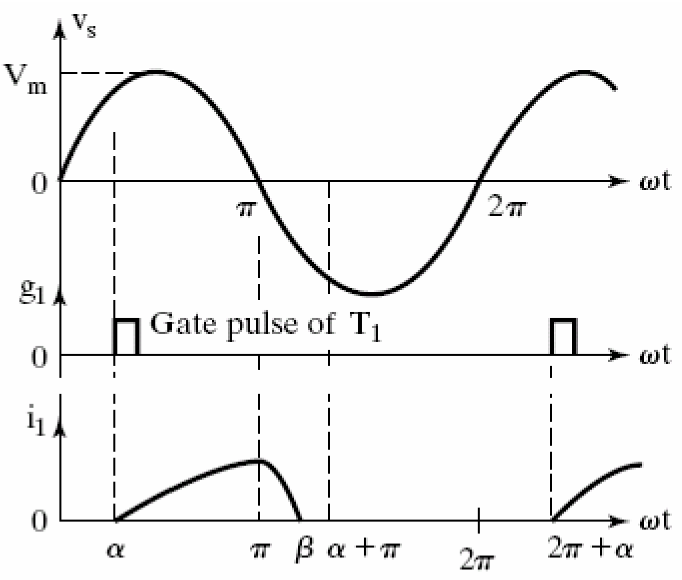
A single phase half wave controlled rectifier circuit with an RL load using a thyristor T_1 (T_1 is an SCR) is shown in the figure below.



The thyristor T_1 is forward biased during the positive half cycle of input supply. Let us assume that T_1 is triggered at $\omega t = \alpha$, by applying a suitable gate trigger pulse to T_1 during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when T_1 is ON. The load current i_O flows through the thyristor T_1 and through the load in the downward direction. This load current pulse flowing through T_1 can be considered as the positive current pulse. Due to the inductance in the load, the load current i_O flowing through T_1 would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative. A phase shift appears between the load voltage and the load current waveforms, due to the load inductance.

The thyristor T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta$, where β is referred to as the Extinction angle, (the value of ωt) at which the load current falls to zero. The extinction angle β is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor T_1 conducts from $\omega t = \alpha$ to β. The conduction angle of T_1 is $\delta = (\beta - \alpha)$, which depends on the delay angle α and the load impedance angle ϕ. The waveforms of the input supply voltage, the gate trigger pulse of T_1, the thyristor current, the load current and the load voltage waveforms appear as shown in the figure below.



$$i_1 = i_O = i_S$$

Fig.: Input supply voltage & Thyristor current waveforms

β is the extinction angle which depends upon the load inductance value.

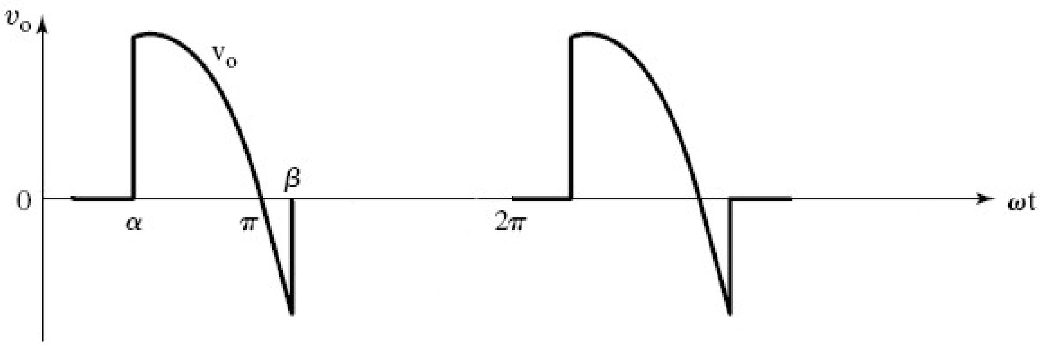


Fig.: Output (load) voltage waveform of a single phase half wave controlled rectifier with RL load

From β to 2π, the thyristor remains cut-off as it is reverse biased and behaves as an open switch. The thyristor current and the load current are zero and the output voltage also remains at zero during the non conduction time interval between β to 2π. In the next cycle the thyristor is triggered again at a phase angle of $(2\pi + \alpha)$, and the same operation repeats.

TO DERIVE AN EXPRESSION FOR THE OUTPUT (INDUCTIVE LOAD) CURRENT, DURING $\omega t = \alpha$ to β WHEN THYRISTOR T_1 CONDUCTS

Considering sinusoidal input supply voltage we can write the expression for the supply voltage as

$v_S = V_m \sin \omega t$ = instantaneous value of the input supply voltage.

Let us assume that the thyristor T_1 is triggered by applying the gating signal to T_1 at $\omega t = \alpha$. The load current which flows through the thyristor T_1 during $\omega t = \alpha$ to β can be found from the equation

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \quad ;$$

The solution of the above differential equation gives the general expression for the output load current which is of the form

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}} \quad ;$$

Where $V_m = \sqrt{2}V_S$ = maximum or peak value of input supply voltage.

$$Z = \sqrt{R^2 + \left(\omega L\right)^2} \text{ = Load impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ = Load impedance angle (power factor angle of load).}$$

$$\tau = \frac{L}{R} = \text{ Load circuit time constant.}$$

Therefore the general expression for the output load current is given by the equation

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-R}{L}t} \quad ;$$

The value of the constant A_1 can be determined from the initial condition. i.e. initial value of load current $i_O = 0$, at $\omega t = \alpha$. Hence from the equation for i_O equating i_O to zero and substituting $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) + A_1 e^{\frac{-R}{L}t}$$

Therefore $$A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin\left(\alpha - \phi\right)$$

$$A_1 = \frac{1}{e^{\frac{-R}{L}t}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$A_1 = e^{\frac{+R}{L}t}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$A_1 = e^{\frac{R(\omega t)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

By substituting $\omega t = \alpha$, we get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

Substituting the value of constant A_1 from the above equation into the expression for i_O , we obtain

$$i_O = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R}{L}t} e^{\frac{R(\alpha)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right] \;;$$

$$i_O = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R(\omega t)}{\omega L}} e^{\frac{R(\alpha)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

$$i_O = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R}{\omega L}(\omega t - \alpha)} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

Therefore we obtain the final expression for the inductive load current of a single phase half wave controlled rectifier with RL load as

$$i_O = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right] \;; \quad \text{Where } \alpha \le \omega t \le \beta .$$

The above expression also represents the thyristor current i_{T1}, during the conduction time interval of thyristor T_1 from $\omega t = \alpha$ to β .

TO CALCULATE EXTINCTION ANGLE β

The extinction angle β, which is the value of ωt at which the load current i_O falls to zero and T_1 is turned off can be estimated by using the condition that $i_O = 0$, at $\omega t = \beta$

By using the above expression for the output load current, we can write

$$i_O = 0 = \frac{V_m}{Z} \left[\sin(\beta - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\beta - \alpha)} \right]$$

As $\frac{V_m}{Z} \neq 0$, we can write

$$\left[\sin(\beta - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\beta - \alpha)} \right] = 0$$

Therefore we obtain the expression

$$\sin(\beta-\phi)=\sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}$$

The extinction angle β can be determined from this transcendental equation by using the iterative method of solution (trial and error method). After β is calculated, we can determine the thyristor conduction angle $\delta=(\beta-\alpha)$.

β is the extinction angle which depends upon the load inductance value. Conduction angle δ increases as α is decreased for a specific value of β.

Conduction angle $\delta=(\beta-\alpha)$; for a purely resistive load or for an RL load when the load inductance L is negligible the extinction angle $\beta=\pi$ and the conduction angle $\delta=(\pi-\alpha)$

Equations

$v_s = V_m \sin \omega t =$ Input supply voltage

$v_O = v_L = V_m \sin \omega t =$ Output load voltage for $\omega t = \alpha$ to β,

when the thyristor T_1 conducts (T_1 is on).

Expression for the load current (thyristor current): for $\omega t = \alpha$ to β

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] \quad ; \quad \text{Where } \alpha \le \omega t \le \beta .$$

Extinction angle β can be calculated using the equation

$$\sin\left(\beta - \phi\right) = \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

TO DERIVE AN EXPRESSION FOR AVERAGE (DC) LOAD VOLTAGE

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\int_0^{2\pi} v_O . d\left(\omega t\right)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O . d\left(\omega t\right) + \int_{\alpha}^{\beta} v_O . d\left(\omega t\right) + \int_{\beta}^{2\pi} v_O . d\left(\omega t\right)\right] ;$$

$v_O = 0$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π ;

$$\therefore \quad V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} v_O . d\left(\omega t\right)\right]; v_O = V_m \sin \omega t \text{ for } \omega t = \alpha \text{ to } \beta$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin \omega t . d\left(\omega t\right)\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left[-\cos \omega t \Big/_{\alpha}^{\beta}\right] = \frac{V_m}{2\pi}\left(\cos \alpha - \cos \beta\right)$$

$$\therefore \quad V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left(\cos \alpha - \cos \beta\right)$$

Note: During the period $\omega t = \pi$ to β, we can see from the output load voltage waveform that the instantaneous output voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

Average DC Load Current

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\pi R_L}\left(\cos\alpha - \cos\beta\right)$$

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RL LOAD AND FREE WHEELING DIODE

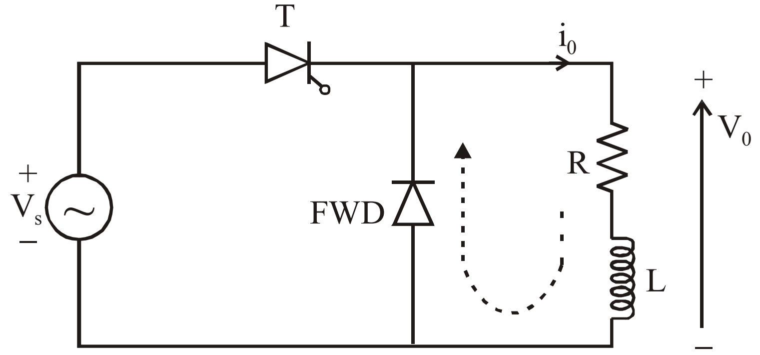
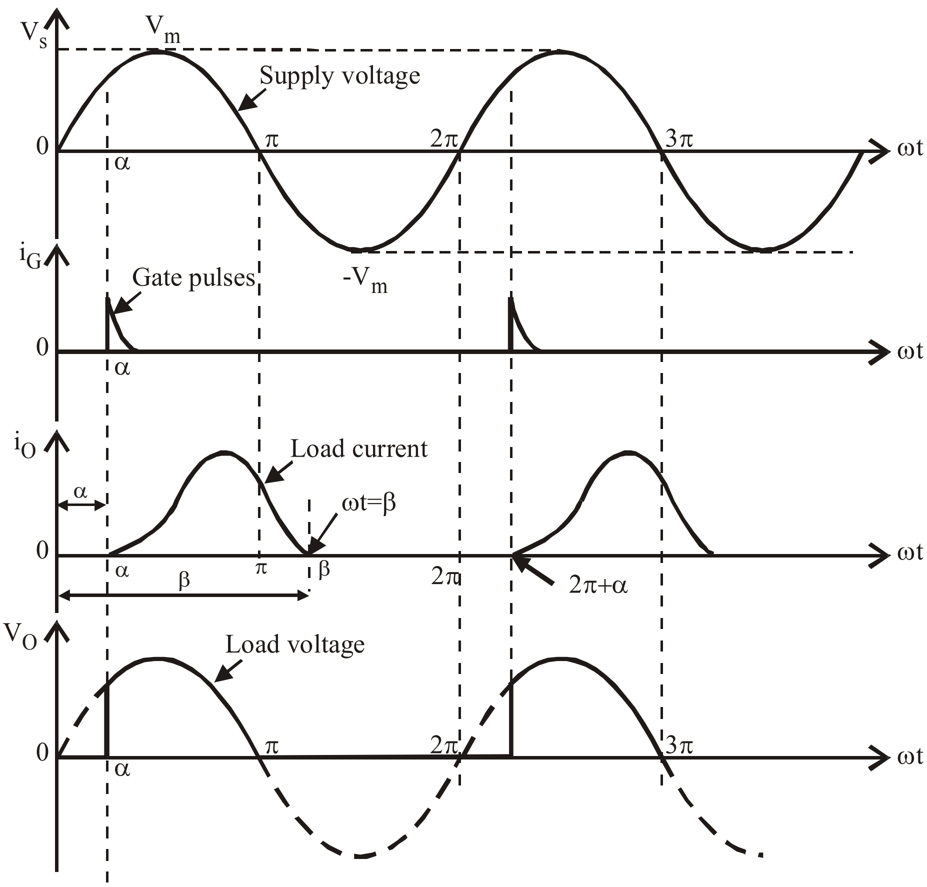


Fig. : Single Phase Half Wave Controlled Rectifier with RL Load and Free Wheeling Diode (FWD)

With a RL load it was observed that the average output voltage reduces. This disadvantage can be overcome by connecting a diode across the load as shown in figure. The diode is called as a *Free Wheeling Diode (FWD)*. The waveforms are shown below.



At $\omega t = \pi$, the source voltage v_S falls to zero and as v_S becomes negative, the free wheeling diode is forward biased. The stored energy in the inductance maintains the load current flow through R, L, and the FWD. Also, as soon as the FWD is forward biased, at $\omega t = \pi$, the SCR becomes reverse biased, the current through it becomes zero and the SCR turns off. During the period $\omega t = \pi$ to β, the load current flows through FWD (free wheeling load current) and decreases exponentially towards zero at $\omega t = \beta$.

Also during this free wheeling time period the load is shorted by the conducting FWD and the load voltage is almost zero, if the forward voltage drop across the conducting FWD is neglected. Thus there is no negative region in the load voltage wave form. This improves the average output voltage.

The average output voltage $V_{dc} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right]$, which is the same as that of a purely resistive load. The output voltage across the load appears similar to the output voltage of a purely resistive load.

The following points are to be noted.

- If the inductance value is not very large, the energy stored in the inductance is able to maintain the load current only upto $\omega t = \beta$, where $\pi < \beta < 2\pi$, well before the next gate pulse and the load current tends to become discontinuous.
- During the conduction period α to π, the load current is carried by the SCR and during the free wheeling period π to β, the load current is carried by the free wheeling diode.
- The value of β depends on the value of R and L and the forward resistance of the FWD. Generally $\pi < \beta < 2\pi$.

If the value of the inductance is very large, the load current does not decrease to zero during the free wheeling time interval and the load current waveform appears as shown in the figure.

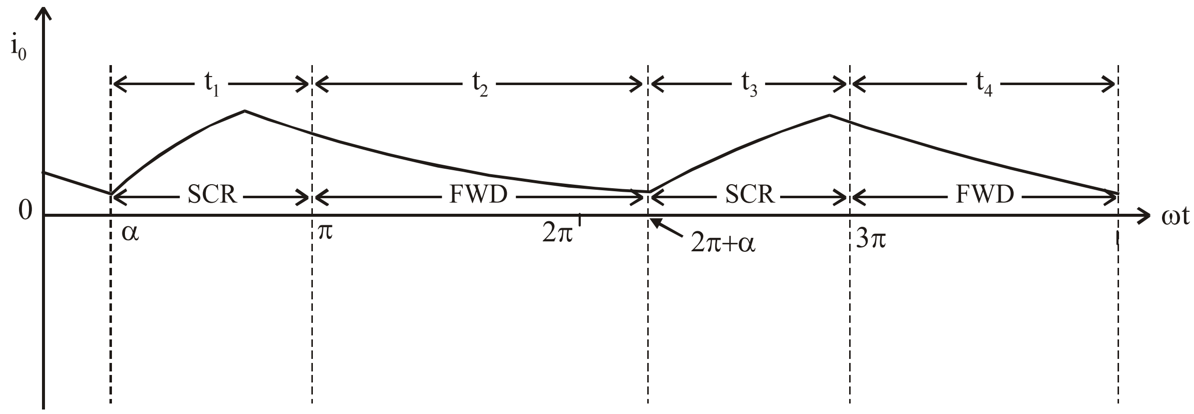


Fig. : Waveform of Load Current in Single Phase Half Wave Controlled Rectifier with a Large Inductance and FWD

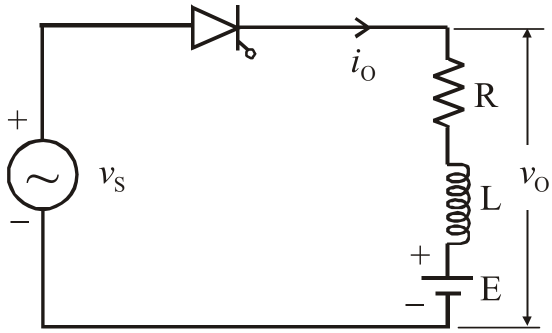
During the periods $t_1, t_3, ...$ the SCR carries the load current and during the periods $t_2, t_4, ...$ the FWD carries the load current.

It is to be noted that

- The load current becomes continuous and the load current does not fall to zero for large value of load inductance.
- The ripple in the load current waveform (the amount of variation in the output load current) decreases.

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH A GENERAL LOAD

A general load consists of R, L and a DC source 'E' in the load circuit



In the half wave controlled rectifier circuit shown in the figure, the load circuit consists of a dc source 'E' in addition to resistance and inductance. When the thyristor is in the cut-off state, the current in the circuit is zero and the cathode will be at a voltage equal to the dc voltage in the load circuit i.e. the cathode potential will be equal to 'E'. The thyristor will be forward biased for anode supply voltage greater than the load dc voltage.

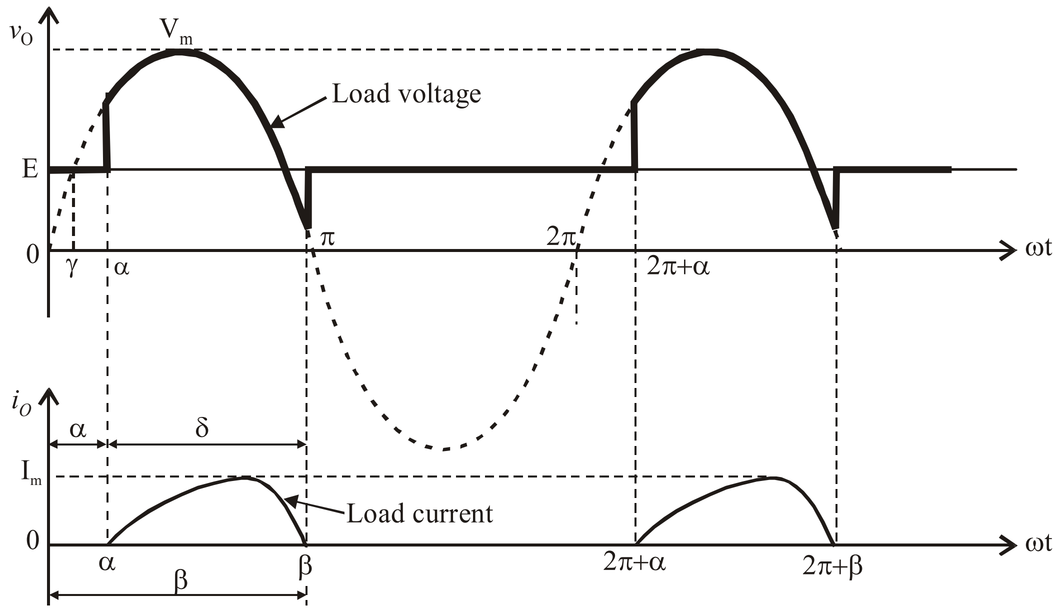
When the supply voltage is less than the dc voltage 'E' in the circuit the thyristor is reverse biased and hence the thyristor cannot conduct for supply voltage less than the load circuit dc voltage.

The value of ωt at which the supply voltage increases and becomes equal to the load circuit dc voltage can be calculated by using the equation $V_m \sin \omega t = E$. If we assume the value of ωt is equal to γ then we can write $V_m \sin \gamma = E$. Therefore γ is calculated as $\gamma = \sin^{-1}\left(\frac{E}{V_m}\right)$.

For trigger angle $\alpha < \gamma$, the thyristor conducts only from $\omega t = \gamma$ to β.

For trigger angle $\alpha > \gamma$, the thyristor conducts from $\omega t = \alpha$ to β.

The waveforms appear as shown in the figure



Equations

$v_S = V_m \sin \omega t =$ Input supply voltage .

$v_O = V_m \sin \omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

Expression for the Load Current

When the thyristor is triggered at a delay angle of α, the equation for the circuit can be written as

$$V_m \sin \omega t = i_O \times R + L\left(\frac{di_O}{dt}\right) + \text{E} \ ; \ \alpha \le \omega t \le \beta$$

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-t}{\tau}}$$

Where

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load Impedance}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{ Load impedance angle}$$

$$\tau = \frac{L}{R} = \text{ Load circuit time constant}$$

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}t}$$

To find the value of the constant 'A' apply the initial condition at $\omega t = \alpha$, load current $i_O = 0$. Equating the general expression for the load current to zero at $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}\times\frac{\alpha}{\omega}}$$

We obtain the value of constant 'A' as

$$A = \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{R}{\omega L}\alpha}$$

Substituting the value of the constant 'A' in the expression for the load current, we get the complete expression for the output load current as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{-R}{\omega L}(\omega t - \alpha)}$$

The Extinction angle β can be calculated from the final condition that the output current $i_O = 0$ at $\omega t = \beta$. By using the above expression we get,

$$i_O = 0 = \frac{V_m}{Z}\sin(\beta - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

To derive an expression for the average or dc load voltage

$$V_{O(dc)} = \frac{1}{2\pi}\int_0^{2\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d(\omega t) + \int_{\alpha}^{\beta} v_O.d(\omega t) + \int_{\beta}^{2\pi} v_O.d(\omega t)\right]$$

$v_O = V_m \sin\omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} E.d(\omega t) + \int_{\alpha}^{\beta} V_m \sin\omega t + \int_{\beta}^{2\pi} E.d(\omega t)\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\omega t)\Big/_0^{\alpha} + V_m(-\cos\omega t)\Big/_{\alpha}^{\beta} + E(\omega t)\Big/_{\beta}^{2\pi}\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\alpha - 0) - V_m(\cos\beta - \cos\alpha) + E(2\pi - \beta)\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[(\cos\alpha - \cos\beta)\right] + \frac{E}{2\pi}(2\pi - \beta + \alpha)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) + \left[\frac{2\pi - (\beta - \alpha)}{2\pi}\right]E$$

Conduction angle of thyristor $\delta = (\beta - \alpha)$

RMS Output Voltage can be calculated by using the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2 . d(\omega t)\right]}$$

DISADVANTAGES OF SINGLE PHASE HALF WAVE CONTROLLED RECTIFIERS

Single phase half wave controlled rectifier gives

- Low dc output voltage.
- Low dc output power and lower efficiency.
- Higher ripple voltage & ripple current.
- Higher ripple factor.
- Low transformer utilization factor.
- The input supply current waveform has a dc component which can result in dc saturation of the transformer core.

Single phase half wave controlled rectifiers are rarely used in practice as they give low dc output and low dc output power. They are only of theoretical interest.

The above disadvantages of a single phase half wave controlled rectifier can be over come by using a full wave controlled rectifier circuit. Most of the practical converter circuits use full wave controlled rectifiers.

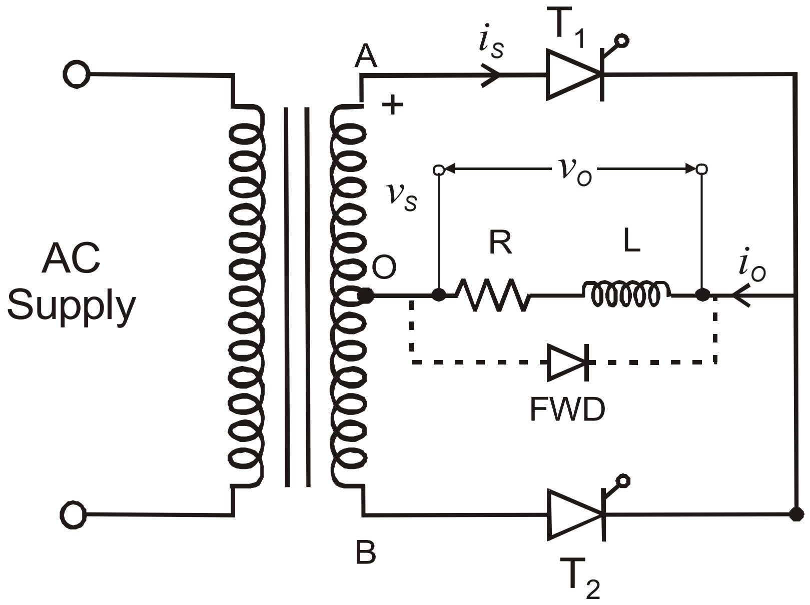
SINGLE PHASE FULL WAVE CONTROLLED RECTIFIERS

Single phase full wave controlled rectifier circuit combines two half wave controlled rectifiers in one single circuit so as to provide two pulse output across the load. Both the half cycles of the input supply are utilized and converted into a uni-directional output current through the load so as to produce a two pulse output waveform. Hence a full wave controlled rectifier circuit is also referred to as a two pulse converter.

Single phase full wave controlled rectifiers are of various types

- Single phase full wave controlled rectifier using a center tapped transformer (two pulse converter with mid point configuration).
- Single phase full wave bridge controlled rectifier
    - Half controlled bridge converter (semi converter).
    - Fully controlled bridge converter (full converter).

SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER USING A CENTER TAPPED TRANSFORMER



v_S = Supply Voltage across the upper half of the transformer secondary winding

$$v_S = v_{AO} = V_m \sin \omega t$$

$v_{BO} = -v_{AO} = -V_m \sin \omega t =$ supply voltage across the lower half of the transformer secondary winding.

This type of full wave controlled rectifier requires a center tapped transformer and two thyristors T_1 and T_2. The input supply is fed through the mains supply transformer, the primary side of the transformer is connected to the ac line voltage which is available (normally the primary supply voltage is 230V RMS ac supply voltage at 50Hz supply frequency in India). The secondary side of the transformer has three lines and the center point of the transformer (center line) is used as the reference point to measure the input and output voltages.

The upper half of the secondary winding and the thyristor T_1 along with the load act as a half wave controlled rectifier, the lower half of the secondary winding and the thyristor T_2 with the common load act as the second half wave controlled rectifier so as to produce a full wave load voltage waveform.

There are two types of operations possible.

- Discontinuous load current operation, which occurs for a purely resistive load or an RL load with low inductance value.
- Continuous load current operation which occurs for an RL type of load with large load inductance.

Discontinuous Load Current Operation (for low value of load inductance)

Generally the load current is discontinuous when the load is purely resistive or when the RL load has a low value of inductance.

During the positive half cycle of input supply, when the upper line of the secondary winding is at a positive potential with respect to the center point 'O' the thyristor T_1 is forward biased and it is triggered at a delay angle of α. The load current flows through the thyristor T_1, through the load and through the upper part of the secondary winding, during the period α to β, when the thyristor T_1 conducts.

The output voltage across the load follows the input supply voltage that appears across the upper part of the secondary winding from $\omega t = \alpha$ to β. The load current through the thyristor T_1 decreases and drops to zero at $\omega t = \beta$, where $\beta > \pi$ for RL type of load and the thyristor T_1 naturally turns off at $\omega t = \beta$.

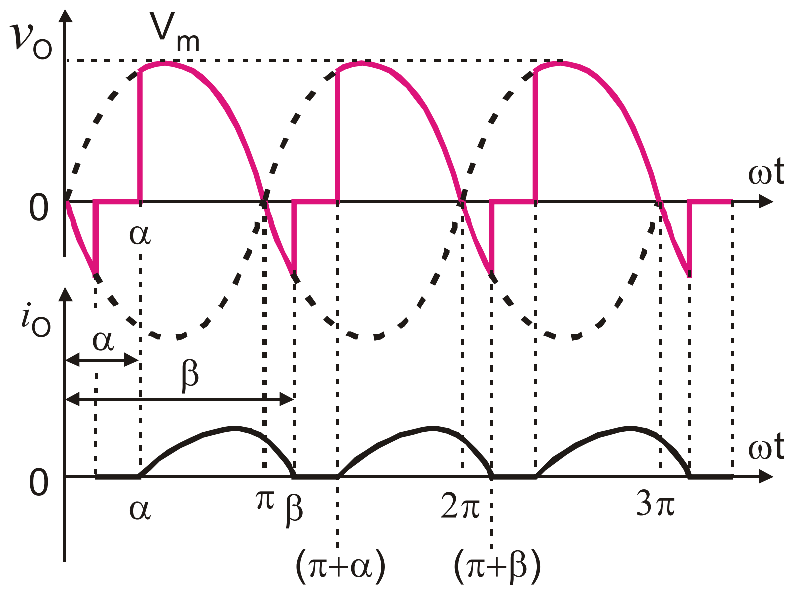


Fig.: Waveform for Discontinuous Load Current Operation without FWD

During the negative half cycle of the input supply the voltage at the supply line 'A' becomes negative whereas the voltage at line 'B' (at the lower side of the secondary winding) becomes positive with respect to the center point 'O'. The thyristor T_2 is forward biased during the negative half cycle and it is triggered at a delay angle of $(\pi + \alpha)$. The current flows through the thyristor T_2, through the load, and through the lower part of the secondary winding when T_2 conducts during the negative half cycle the load is connected to the lower half of the secondary winding when T_2 conducts.

For purely resistive loads when L = 0, the extinction angle $\beta = \pi$. The load current falls to zero at $\omega t = \beta = \pi$, when the input supply voltage falls to zero at $\omega t = \pi$. The load current and the load voltage waveforms are in phase and there is no phase shift between the load voltage and the load current waveform in the case of a purely resistive load.

For low values of load inductance the load current would be discontinuous and the extinction angle $\beta > \pi$ but $\beta < (\pi + \alpha)$.

For large values of load inductance the load current would be continuous and does not fall to zero. The thyristor T_1 conducts from α to $(\pi + \alpha)$, until the next thyristor T_2 is triggered. When T_2 is triggered at $\omega t = (\pi + \alpha)$, the thyristor T_1 will be reverse biased and hence T_1 turns off.

TO DERIVE AN EXPRESSION FOR THE DC OUTPUT VOLTAGE OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH RL LOAD (WITHOUT FREE WHEELING DIODE (FWD))

The average or dc output voltage of a full-wave controlled rectifier can be calculated by finding the average value of the output voltage waveform over one output cycle (i.e., π radians) and note that the output pulse repetition time is $\frac{T}{2}$ seconds where T represents the input supply time period and $T = \frac{1}{f}$; where f = input supply frequency.

Assuming the load inductance to be small so that $\beta > \pi$, $\beta < (\pi + \alpha)$ we obtain discontinuous load current operation. The load current flows through T_1 form $\omega t = \alpha$ to β, where α is the trigger angle of thyristor T_1 and β is the extinction angle where the load current through T_1 falls to zero at $\omega t = \beta$. Therefore the average or dc output voltage can be obtained by using the expression

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi} \int_{\omega t = \alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t = \alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

Therefore $V_{O(dc)} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$**, for discontinuous load current operation,** $\pi < \beta < (\pi + \alpha)$**.**

When the load inductance is small and negligible that is $L \approx 0$, the extinction angle $\beta = \pi$ radians. Hence the average or dc output voltage for resistive load is obtained as

$$V_{O(dc)} = \frac{V_m}{\pi} (\cos \alpha - \cos \pi) \quad ; \; \cos \pi = -1$$

$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - (-1)\right)$$

$$V_{O(dc)} = \frac{V_m}{\pi}\left(1+\cos\alpha\right) \text{ ; for resistive load, when } L \approx 0$$

THE EFFECT OF LOAD INDUCTANCE

Due to the presence of load inductance the output voltage reverses and becomes negative during the time period $\omega t = \pi$ to β. This reduces the dc output voltage. To prevent this reduction of dc output voltage due to the negative region in the output load voltage waveform, we can connect a free wheeling diode across the load. The output voltage waveform and the dc output voltage obtained would be the same as that for a full wave controlled rectifier with resistive load.

When the Free wheeling diode (FWD) is connected across the load

When T_1 is triggered at $\omega t = \alpha$, during the positive half cycle of the input supply the FWD is reverse biased during the time period $\omega t = \alpha$ to π. FWD remains reverse biased and cut-off from $\omega t = \alpha$ to π. The load current flows through the conducting thyristor T_1, through the RL load and through upper half of the transformer secondary winding during the time period α to π.

At $\omega t = \pi$, when the input supply voltage across the upper half of the secondary winding reverses and becomes negative the FWD turns-on. The load current continues to flow through the FWD from $\omega t = \pi$ to β.

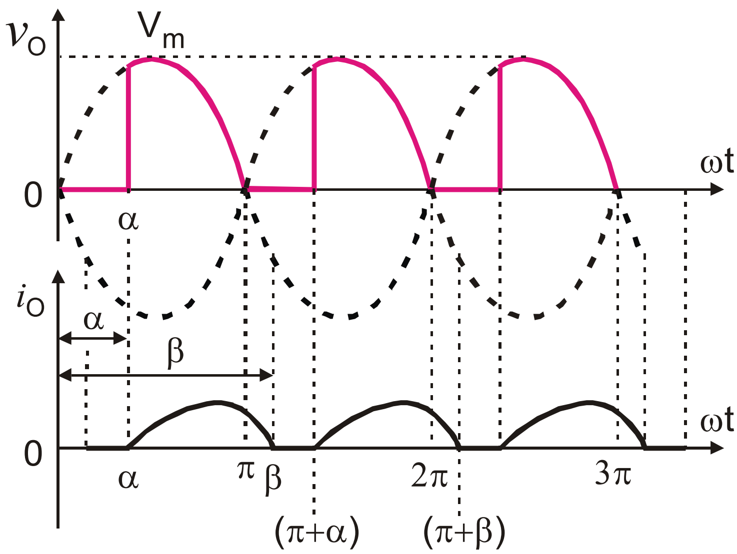


Fig.: Waveform for Discontinuous Load Current Operation with FWD

EXPRESSION FOR THE DC OUTPUT VOLTAGE OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH RL LOAD AND FWD

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O .d(\omega t)$$

Thyristor T_1 is triggered at $\omega t = \alpha$. T_1 conducts from $\omega t = \alpha$ to π

Output voltage $v_O = V_m \sin \omega t$; for $\omega t = \alpha \ to \ \pi$

FWD conducts from $\omega t = \pi$ to β and $v_O \approx 0$ during discontinuous load current

Therefore $$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t .d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \quad ; \quad \cos \pi = -1$$

Therefore $$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

The DC output voltage V_{dc} is same as the DC output voltage of a single phase full wave controlled rectifier with resistive load. Note that the dc output voltage of a single phase full wave controlled rectifier is two times the dc output voltage of a half wave controlled rectifier.

CONTROL CHARACTERISTICS OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH R LOAD OR RL LOAD WITH FWD

The control characteristic can be obtained by plotting the dc output voltage V_{dc} versus the trigger angle α.

The average or dc output voltage of a single phase full wave controlled rectifier circuit with R load or RL load with FWD is calculated by using the equation

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

V_{dc} can be varied by varying the trigger angle α from 0 to 180^0. (i.e., the range of trigger angle α is from 0 to π radians).

Maximum dc output voltage is obtained when $\alpha = 0$

$$V_{dc(\max)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos 0) = \frac{2V_m}{\pi}$$

Therefore $V_{dc(\max)} = V_{dc} = \dfrac{2V_m}{\pi}$ for a single phase full wave controlled rectifier.

Normalizing the dc output voltage with respect to its maximum value, we can write the normalized dc output voltage as

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

$$V_{dcn} = V_n = \frac{\frac{V_m}{\pi}\left(1+\cos\alpha\right)}{\left(\frac{2V_m}{\pi}\right)} = \frac{1}{2}\left(1+\cos\alpha\right)$$

Therefore $\quad V_{dcn} = V_n = \frac{1}{2}\left(1+\cos\alpha\right) = \frac{V_{dc}}{V_{dm}}$

$$V_{dc} = \frac{1}{2}\left(1+\cos\alpha\right)V_{dm}$$

Trigger angle α in degrees	$V_{O(dc)}$	**Normalized dc output voltage V_n**
0	$V_{dm} = \frac{2V_m}{\pi} = 0.636619V_m$	1
30^0	$0.593974\ V_m$	0.9330
60^0	$0.47746\ V_m$	0.75
90^0	$0.3183098\ V_m$	0.5
120^0	$0.191549\ V_m$	0.25
150^0	$0.04264\ V_m$	0.06698
180^0	0	0

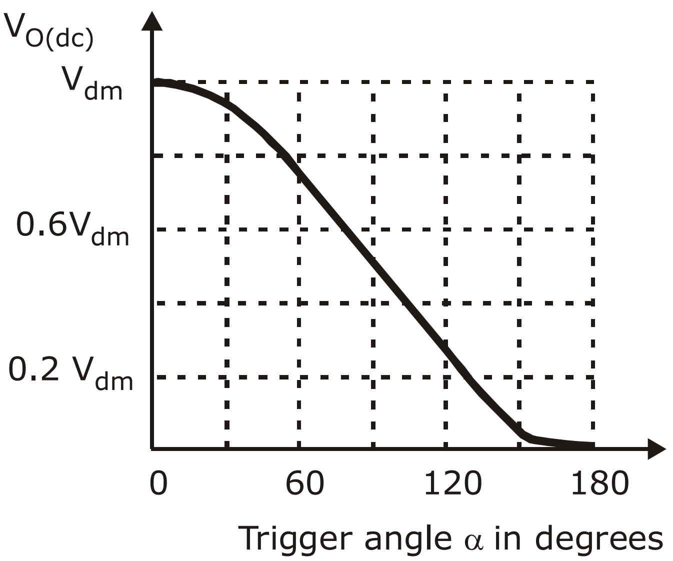


Fig.: Control characteristic of a single phase full wave controlled rectifier with R load or RL load with FWD

CONTINUOUS LOAD CURRENT OPERATION (WITHOUT FWD)

For large values of load inductance the load current flows continuously without decreasing and falling to zero and there is always a load current flowing at any point of time. This type of operation is referred to as continuous current operation.

Generally the load current is continuous for large load inductance and for low trigger angles.

The load current is discontinuous for low values of load inductance and for large values of trigger angles.

The waveforms for continuous current operation are as shown.

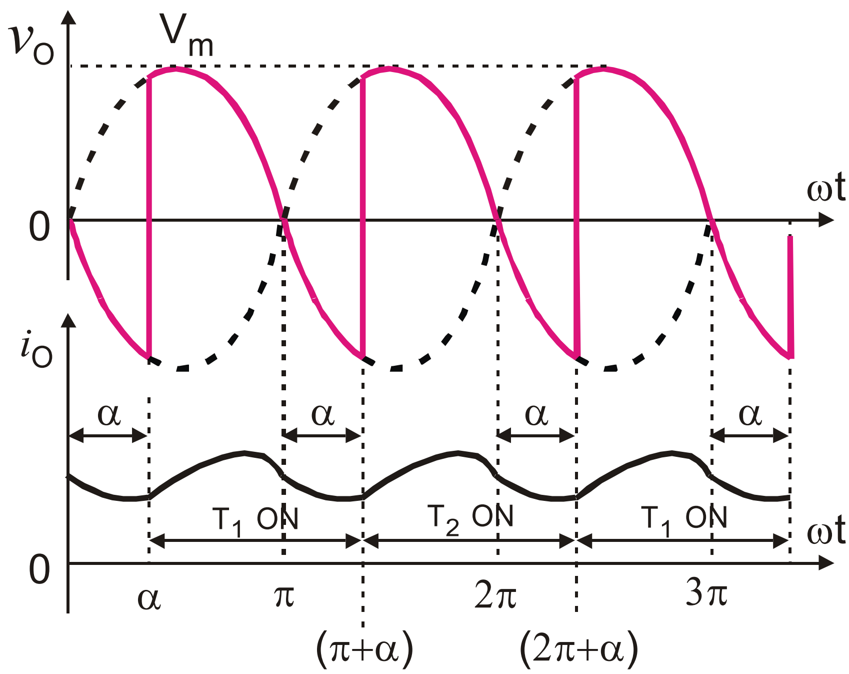


Fig.: Load voltage and load current waveform of a single phase full wave controlled rectifier with RL load & without FWD for continuous load current operation

In the case of continuous current operation the thyristor T_1 which is triggered at a delay angle of α, conducts from $\omega t = \alpha$ to $(\pi + \alpha)$. Output voltage follows the input supply voltage across the upper half of the transformer secondary winding $v_O = v_{AO} = V_m \sin \omega t$.

The next thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$, during the negative half cycle input supply. As soon as T_2 is triggered at $\omega t = (\pi + \alpha)$, the thyristor T_1 will be reverse biased and T_1 turns off due to natural commutation (ac line commutation). The load current flows through the thyristor T_2 from $\omega t = (\pi + \alpha)$ to $(2\pi + \alpha)$.

Output voltage across the load follows the input supply voltage across the lower half of the transformer secondary winding $v_O = v_{BO} = -V_m \sin \omega t$.

Each thyristor conducts for π radians $\left(180^0\right)$ in the case of continuous current operation.

TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH LARGE LOAD INDUCTANCE ASSUMING CONTINUOUS LOAD CURRENT OPERATION.

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{(\pi+\alpha)} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{(\pi+\alpha)} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{(\pi+\alpha)} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[\cos \alpha - \cos(\pi + \alpha) \right] \quad ; \qquad \cos(\pi + \alpha) = -\cos \alpha$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[\cos \alpha + \cos \alpha \right]$$

$$\therefore \qquad V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos \alpha$$

The above equation can be plotted to obtain the control characteristic of a single phase full wave controlled rectifier with RL load assuming continuous load current operation.

Normalizing the dc output voltage with respect to its maximum value, the normalized dc output voltage is given by

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dc(\max)}} = \frac{\frac{2V_m}{\pi}(\cos \alpha)}{\frac{2V_m}{\pi}} = \cos \alpha$$

Therefore $\qquad V_{dcn} = V_n = \cos \alpha$

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$	

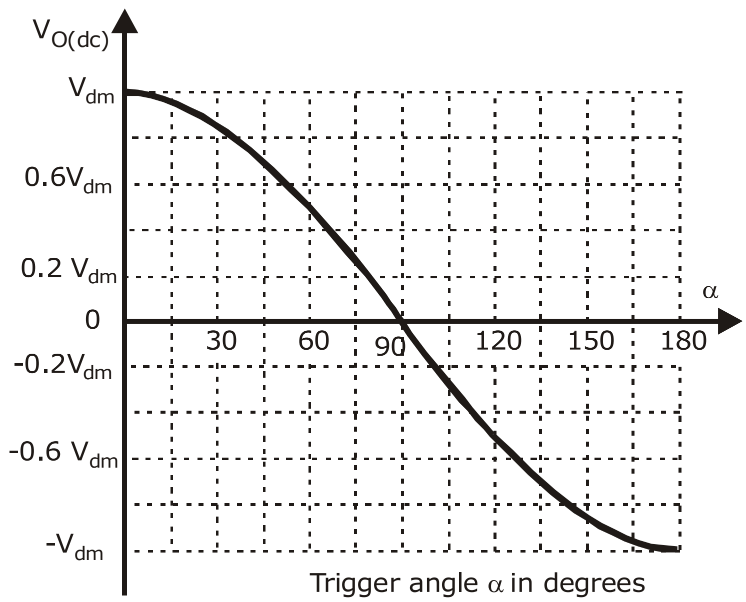


Fig.: Control Characteristic

We notice from the control characteristic that by varying the trigger angle α we can vary the output dc voltage across the load. Thus it is possible to control the dc output voltage by changing the trigger angle α. For trigger angle α in the range of 0 to 90 degrees $\left(i.e.,\ 0 \le \alpha \le 90^0\right)$, V_{dc} is positive and the circuit operates as a controlled rectifier to convert ac supply voltage into dc output power which is fed to the load.

For trigger angle $\alpha > 90^0, \cos\alpha$ becomes negative and as a result the average dc output voltage V_{dc} becomes negative, but the load current flows in the same positive direction. Hence the output power becomes negative. This means that the power flows from the load circuit to the input ac source. This is referred to as *line commutated inverter operation*. During the inverter mode operation for $\alpha > 90^0$ the load energy can be fed back from the load circuit to the input ac source.

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE

The rms value of the output voltage is calculated by using the equation

$$V_{O(RMS)} = \left[\frac{2}{2\pi}\int_{\alpha}^{(\pi+\alpha)} v_O^2.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{1}{\pi}\int_{\alpha}^{(\pi+\alpha)} V_m^2 \sin^2 \omega t.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{\pi}\int_{\alpha}^{(\pi+\alpha)} \sin^2 \omega t.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{\pi}\int_{\alpha}^{(\pi+\alpha)} \frac{(1-\cos 2\omega t)}{2}.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{\int_{\alpha}^{(\pi+\alpha)} d(\omega t) - \int_{\alpha}^{(\pi+\alpha)} \cos 2\omega t.d(\omega t)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{(\omega t)\Big/_{\alpha}^{(\pi+\alpha)} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{(\pi+\alpha)}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{(\pi+\alpha-\alpha) - \left(\frac{\sin 2(\pi+\alpha) - \sin 2\alpha}{2}\right)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{(\pi) - \left(\frac{\sin 2\pi\times\cos 2\alpha + \cos 2\pi\times\sin 2\alpha - \sin 2\alpha}{2}\right)\right\}\right]^{\frac{1}{2}}$$

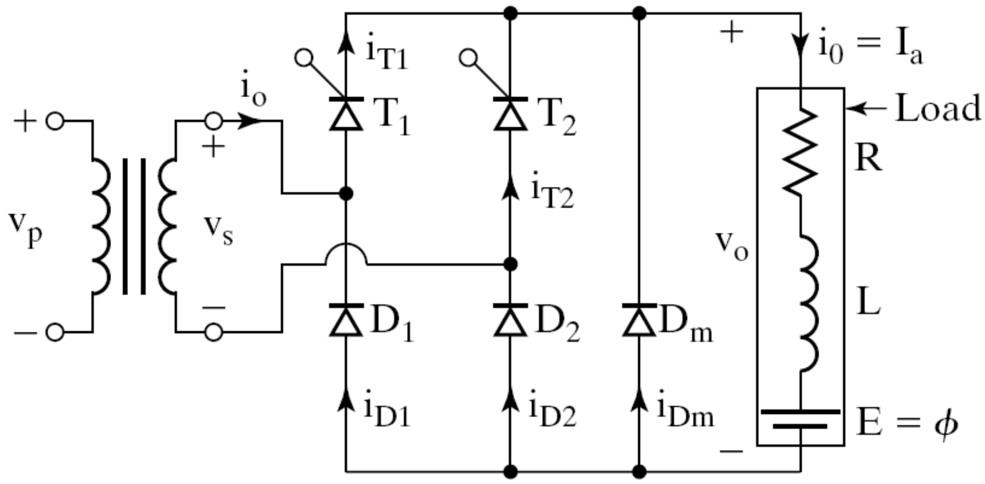
$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{(\pi) - \left(\frac{0 + \sin 2\alpha - \sin 2\alpha}{2}\right)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}(\pi)\right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{2}}$$

Therefore

$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}$; The rms output voltage is same as the input rms supply voltage.

SINGLE PHASE SEMICONVERTERS



Errata: Consider diode D_2 as D_1 in the figure and diode D_1 as D_2

Single phase semi-converter circuit is a full wave half controlled bridge converter which uses two thyristors and two diodes connected in the form of a full wave bridge configuration.

The two thyristors are controlled power switches which are turned on one after the other by applying suitable gating signals (gate trigger pulses). The two diodes are uncontrolled power switches which turn-on and conduct one after the other as and when they are forward biased.

The circuit diagram of a single phase semi-converter (half controlled bridge converter) is shown in the above figure with highly inductive load and a dc source in the load circuit. When the load inductance is large the load current flows continuously and we can consider the continuous load current operation assuming constant load current, with negligible current ripple (i.e., constant and ripple free load current operation).

The ac supply to the semiconverter is normally fed through a mains supply transformer having suitable turns ratio. The transformer is suitably designed to supply the required ac supply voltage (secondary output voltage) to the converter.

During the positive half cycle of input ac supply voltage, when the transformer secondary output line 'A' is positive with respect to the line 'B' the thyristor T_1 and the diode D_1 are both forward biased. The thyristor T_1 is triggered at $\omega t = \alpha$; $(0 \le \alpha \le \pi)$ by applying an appropriate gate trigger signal to the gate of T_1. The current in the circuit flows through the secondary line 'A', through T_1 , through the load in the downward direction, through diode D_1 back to the secondary line 'B'.

T_1 and D_1 conduct together from $\omega t = \alpha$ to π and the load is connected to the input ac supply. The output load voltage follows the input supply voltage (the secondary output voltage of the transformer) during the period $\omega t = \alpha$ to π .

At $\omega t = \pi$, the input supply voltage decreases to zero and becomes negative during the period $\omega t = \pi$ to $(\pi + \alpha)$. The free wheeling diode D_m across the load becomes forward biased and conducts during the period $\omega t = \pi$ to $(\pi + \alpha)$.

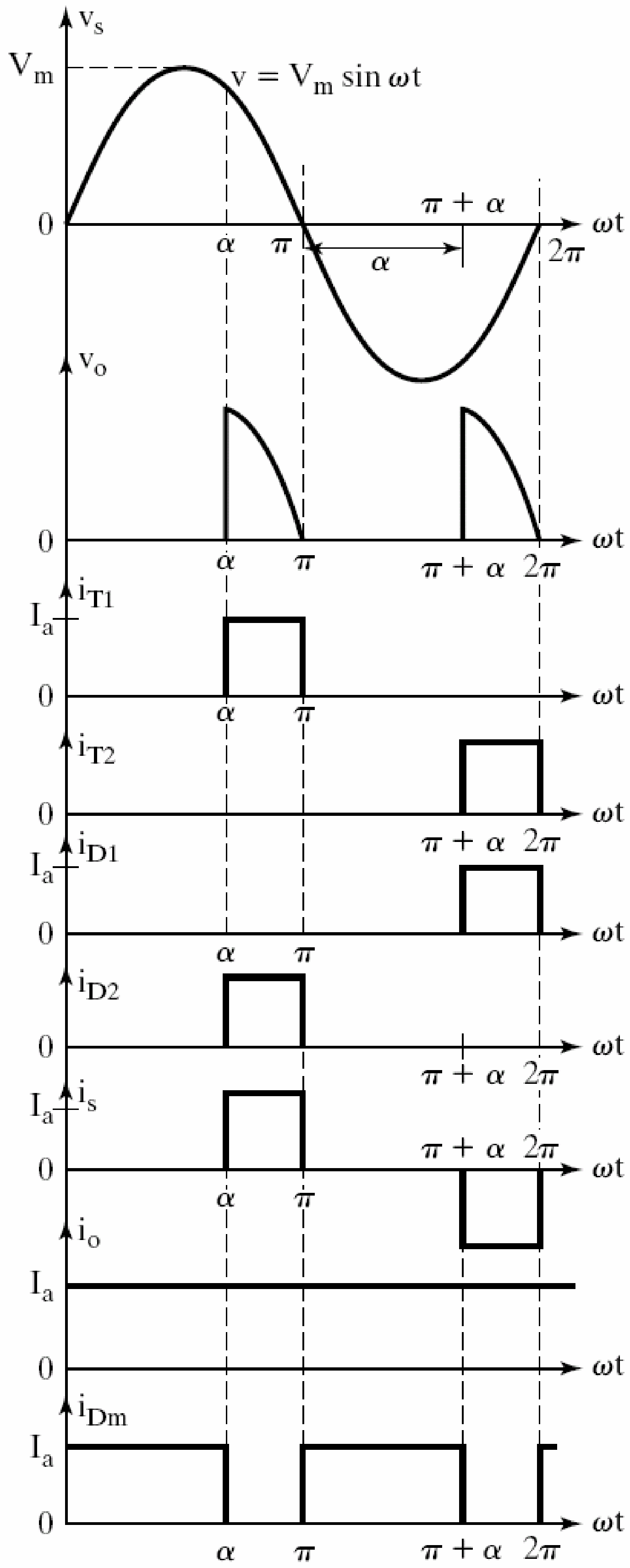


Fig:. Waveforms of single phase semi-converter for RLE load and constant load current for $\alpha > 90^0$

The load current is transferred from T_1 and D_1 to the FWD D_m. T_1 and D_1 are turned off. The load current continues to flow through the FWD D_m. The load current free wheels (flows continuously) through the FWD during the free wheeling time period π to $(\pi+\alpha)$.

During the negative half cycle of input supply voltage the secondary line ‘A’ becomes negative with respect to line ‘B’. The thyristor T_2 and the diode D_2 are both forward biased. T_2 is triggered at $\omega t=(\pi+\alpha)$, during the negative half cycle. The FWD is reverse biased and turns-off as soon as T_2 is triggered. The load current continues to flow through T_2 and D_2 during the period $\omega t=(\pi+\alpha)$ to 2π

TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF A SINGLE PHASE SEMI-CONVERTER

The average output voltage can be found from

$$V_{dc}=\frac{2}{2\pi}\int_{\alpha}^{\pi}V_m\sin\omega t.d(\omega t)$$

$$V_{dc}=\frac{2V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$V_{dc}=\frac{V_m}{\pi}\left[-\cos\pi+\cos\alpha\right]\ ;\ \cos\pi=-1$$

Therefore $$V_{dc}=\frac{V_m}{\pi}\left[1+\cos\alpha\right]$$

V_{dc} can be varied from $\frac{2V_m}{\pi}$ to 0 by varying α from 0 to π.

The maximum average output voltage is

$$V_{dc(\max)}=V_{dm}=\frac{2V_m}{\pi}$$

Normalizing the average output voltage with respect to its maximum value

$$V_{dcn}=V_n=\frac{V_{dc}}{V_{dm}}=0.5\left(1+\cos\alpha\right)$$

The output control characteristic can be plotted by using the expression for V_{dc}

TO DERIVE AN EXPRESSION FOR THE RMS OUTPUT VOLTAGE OF A SINGLE PHASE SEMI-CONVERTER

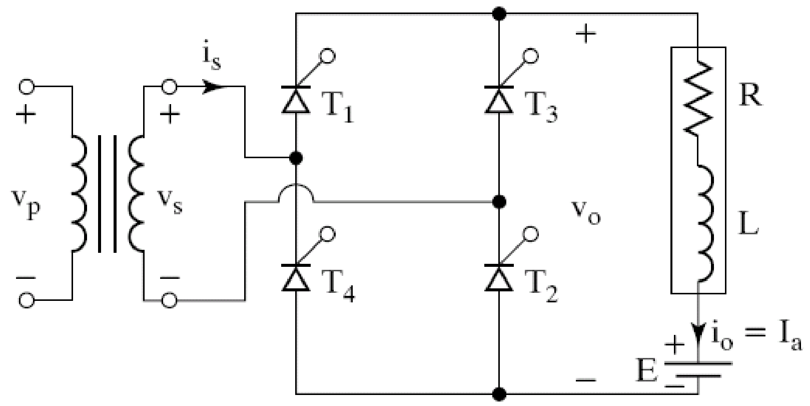
The rms output voltage is found from

$$V_{O(RMS)} = \left[\frac{2}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t.d\left(\omega t\right) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi} \int_{\alpha}^{\pi} \left(1 - \cos 2\omega t\right).d\left(\omega t\right) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

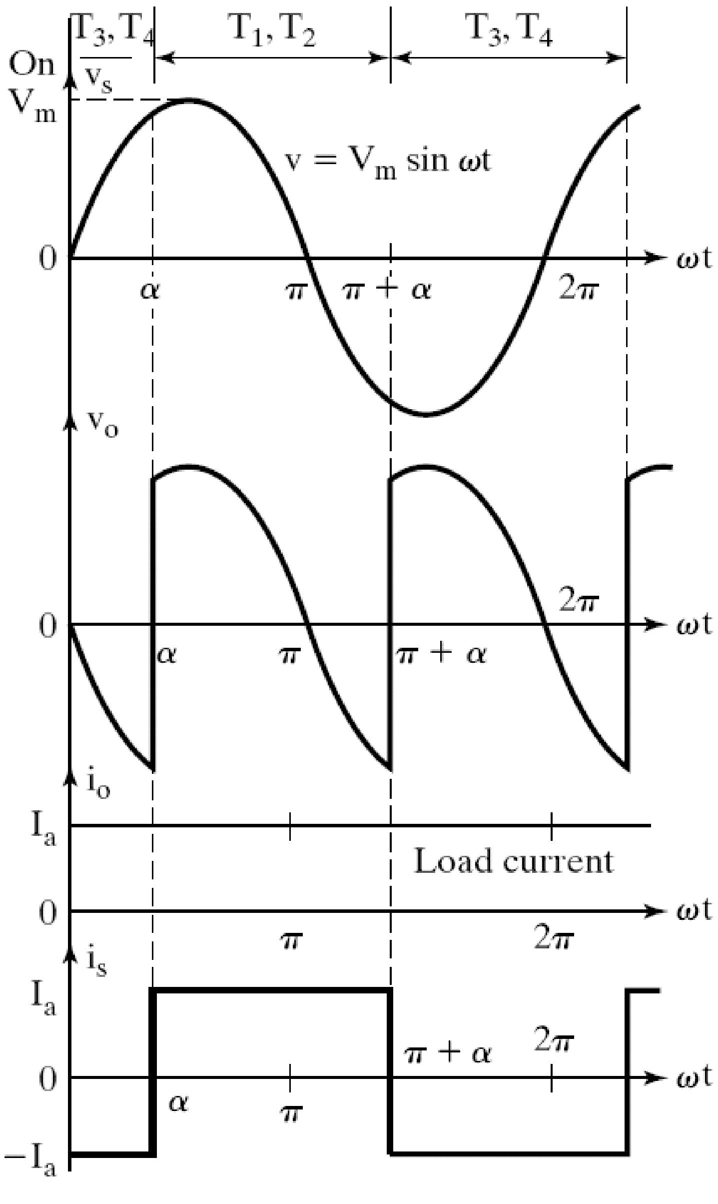
SINGLE PHASE FULL CONVERTER (FULLY CONTROLLED BRIDGE CONVERTER)



The circuit diagram of a single phase fully controlled bridge converter is shown in the figure with a highly inductive load and a dc source in the load circuit so that the load current is continuous and ripple free (constant load current operation).

The fully controlled bridge converter consists of four thyristors T_1, T_2, T_3 and T_4 connected in the form of full wave bridge configuration as shown in the figure. Each thyristor is controlled and turned on by its gating signal and naturally turns off when a reverse voltage appears across it. During the positive half cycle when the upper line of the transformer secondary winding is at a positive potential with respect to the lower end the thyristors T_1 and T_2 are forward biased during the time interval $\omega t = 0$ to π. The thyristors T_1 and T_2 are triggered simultaneously $\omega t = \alpha$; $\left(0 \le \alpha \le \pi\right)$, the load is connected to the input supply through the conducting thyristors T_1 and T_2. The output voltage across the load follows the input supply voltage and hence output voltage $v_O = V_m \sin \omega t$. Due to the inductive load T_1 and T_2 will continue to conduct beyond $\omega t = \pi$, even though the input voltage becomes negative. T_1 and T_2 conduct together during the time period α to $(\pi+\alpha)$, for a time duration of π radians (conduction angle of each thyristor = 180^0)

During the negative half cycle of input supply voltage for $\omega t=\pi$ to 2π the thyristors T_3 and T_4 are forward biased. T_3 and T_4 are triggered at $\omega t=(\pi+\alpha)$. As soon as the thyristors T_3 and T_4 are triggered a reverse voltage appears across the thyristors T_1 and T_2 and they naturally turn-off and the load current is transferred from T_1 and T_2 to the thyristors T_3 and T_4. The output voltage across the load follows the supply voltage and $v_O=-V_m\sin\omega t$ during the time period $\omega t=(\pi+\alpha)$ to $(2\pi+\alpha)$. In the next positive half cycle when T_1 and T_2 are triggered, T_3 and T_4 are reverse biased and they turn-off. The figure shows the waveforms of the input supply voltage, the output load voltage, the constant load current with negligible ripple and the input supply current.



During the time period $\omega t = \alpha$ to π, the input supply voltage v_S and the input supply current i_S are both positive and the power flows from the supply to the load. The converter operates in the rectification mode during $\omega t = \alpha$ to π.

During the time period $\omega t = \pi$ to $(\pi + \alpha)$, the input supply voltage v_S is negative and the input supply current i_S is positive and there will be reverse power flow from the load circuit to the input supply. The converter operates in the inversion mode during the time period $\omega t = \pi$ to $(\pi + \alpha)$ and the load energy is fed back to the input source.

The single phase full converter is extensively used in industrial applications up to about 15kW of output power. Depending on the value of trigger angle α, the average output voltage may be either positive or negative and two quadrant operation is possible.

TO DERIVE AN EXPRESSION FOR THE AVERAGE (DC) OUTPUT VOLTAGE

The average (dc) output voltage can be determined by using the expression

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\left[\int_0^{2\pi} v_O.d(\omega t)\right] \quad ;$$

The output voltage waveform consists of two output pulses during the input supply time period between $0 \ \& \ 2\pi$ radians. In the continuous load current operation of a single phase full converter (assuming constant load current) each thyristor conduct for π radians (180^0) after it is triggered. When thyristors T_1 and T_2 are triggered at $\omega t = \alpha$ T_1 and T_2 conduct from α to $(\pi + \alpha)$ and the output voltage follows the input supply voltage. Therefore output voltage $v_O = V_m \sin \omega t$; for $\omega t = \alpha$ to $(\pi + \alpha)$

Hence the average or dc output voltage can be calculated as

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi}\left[\int_\alpha^{\pi+\alpha} V_m \sin \omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\left[\int_\alpha^{\pi+\alpha} V_m \sin \omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[\int_\alpha^{\pi+\alpha} \sin \omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos \omega t\right]_\alpha^{\pi+\alpha}$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos(\pi+\alpha)+\cos\alpha\right] \quad ;$$

$$\cos(\pi+\alpha) = -\cos\alpha$$

Therefore $\quad V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$

The dc output voltage V_{dc} can be varied from a maximum value of $\frac{2V_m}{\pi}$ for $\alpha = 0^0$ to a minimum value of $\frac{-2V_m}{\pi}$ for $\alpha = \pi$ radians $=180^0$

The maximum average dc output voltage is calculated for a trigger angle $\alpha = 0^0$ and is obtained as

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}\times\cos(0) = \frac{2V_m}{\pi}$$

Therefore $\quad V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$

The normalized average output voltage is given by

$$V_{dcn} = V_n = \frac{V_{O(dc)}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

$$V_{dcn} = V_n = \frac{\frac{2V_m}{\pi}\cos\alpha}{\frac{2V_m}{\pi}} = \cos\alpha$$

Therefore $\quad V_{dcn} = V_n = \cos\alpha \quad$; for a single phase full converter assuming continuous and constant load current operation.

CONTROL CHARACTERISTIC OF SINGLE PHASE FULL CONVERTER

The dc output control characteristic can be obtained by plotting the average or dc output voltage V_{dc} versus the trigger angle α

For a single phase full converter the average dc output voltage is given by the equation $V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$

Trigger angle α in degrees	$V_{O(dc)}$	**Remarks**
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	0.866 V_{dm}	
60^0	0.5 V_{dm}	

90^0	$0\ V_{dm}$
120^0	$-0.5\ V_{dm}$
150^0	$-0.866\ V_{dm}$
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$

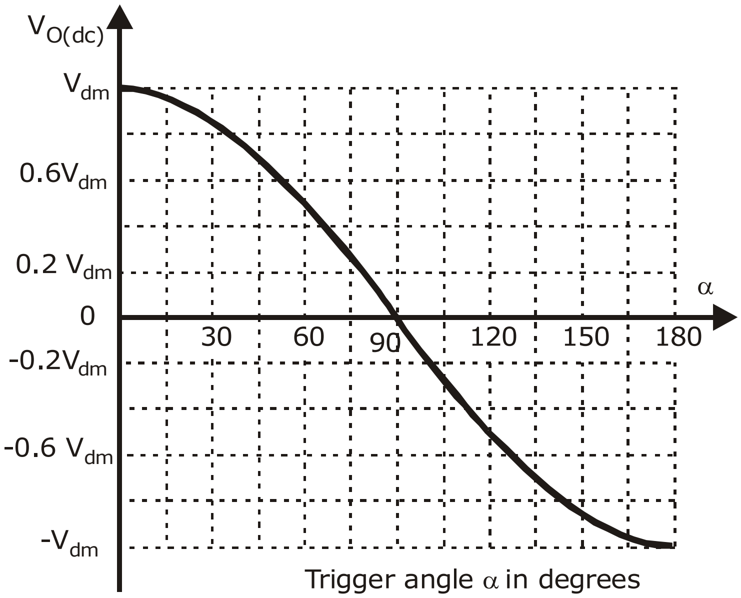
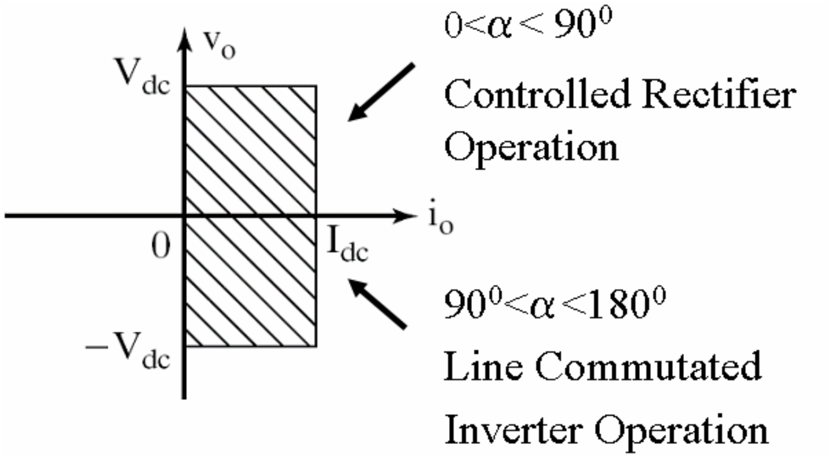


Fig.: Control Characteristic

We notice from the control characteristic that by varying the trigger angle α we can vary the output dc voltage across the load. Thus it is possible to control the dc output voltage by changing the trigger angle α. For trigger angle α in the range of 0 to 90 degrees $\left(i.e.,\ 0 \le \alpha \le 90^0\right)$, V_{dc} is positive and the average dc load current I_{dc} is also positive. The average or dc output power P_{dc} is positive, hence the circuit operates as a controlled rectifier to convert ac supply voltage into dc output power which is fed to the load.

For trigger angle $\alpha > 90^0, \cos\alpha$ becomes negative and as a result the average dc output voltage V_{dc} becomes negative, but the load current flows in the same positive direction i.e., I_{dc} is positive . Hence the output power becomes negative. This means that the power flows from the load circuit to the input ac source. This is referred to as *line commutated inverter operation*. During the inverter mode operation for $\alpha > 90^0$ the load energy can be fed back from the load circuit to the input ac source

TWO QUADRANT OPERATION OF A SINGLE PHASE FULL CONVERTER



The above figure shows the two regions of single phase full converter operation in the V_{dc} versus I_{dc} plane. In the first quadrant when the trigger angle α is less than 90^0, V_{dc} and I_{dc} are both positive and the converter operates as a controlled rectifier and converts the ac input power into dc output power. The power flows from the input source to the load circuit. This is the normal controlled rectifier operation where P_{dc} is positive.

When the trigger angle is increased above 90^0, V_{dc} becomes negative but I_{dc} is positive and the average output power (dc output power) P_{dc} becomes negative and the power flows from the load circuit to the input source. The operation occurs in the fourth quadrant where V_{dc} is negative and I_{dc} is positive. The converter operates as a line commutated inverter.

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF THE OUTPUT VOLTAGE

The rms value of the output voltage is calculated as

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2.d\left(\omega t\right)\right]}$$

The single phase full converter gives two output voltage pulses during the input supply time period and hence the single phase full converter is referred to as a two pulse converter. The rms output voltage can be calculated as

$$V_{O(RMS)} = \sqrt{\frac{2}{2\pi}\left[\int_\alpha^{\pi+\alpha} v_O^2.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_\alpha^{\pi+\alpha} V_m^2 \sin^2 \omega t.d\left(\omega t\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \sin^2 \omega t.d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \frac{(1-\cos 2\omega t)}{2}.d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} d(\omega t) - \int_{\alpha}^{\pi+\alpha} \cos 2\omega t.d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\omega t)\Big/_{\alpha}^{\pi+\alpha} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi+\alpha}\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi+\alpha-\alpha) - \left(\frac{\sin 2(\pi+\alpha) - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi) - \left(\frac{\sin(2\pi+2\alpha) - \sin 2\alpha}{2}\right)\right]} \quad ; \quad \sin(2\pi+2\alpha) = \sin 2\alpha$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi) - \left(\frac{\sin 2\alpha - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}(\pi) - 0} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

Therefore $\quad V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S$

Hence the rms output voltage is same as the rms input supply voltage

The rms thyristor current can be calculated as

Each thyristor conducts for π radians or 180^0 in a single phase full converter operating at continuous and constant load current.

Therefore rms value of the thyristor current is calculated as

$$I_{T(RMS)} = I_{O(RMS)}\sqrt{\frac{\pi}{2\pi}} = I_{O(RMS)}\sqrt{\frac{1}{2}}$$

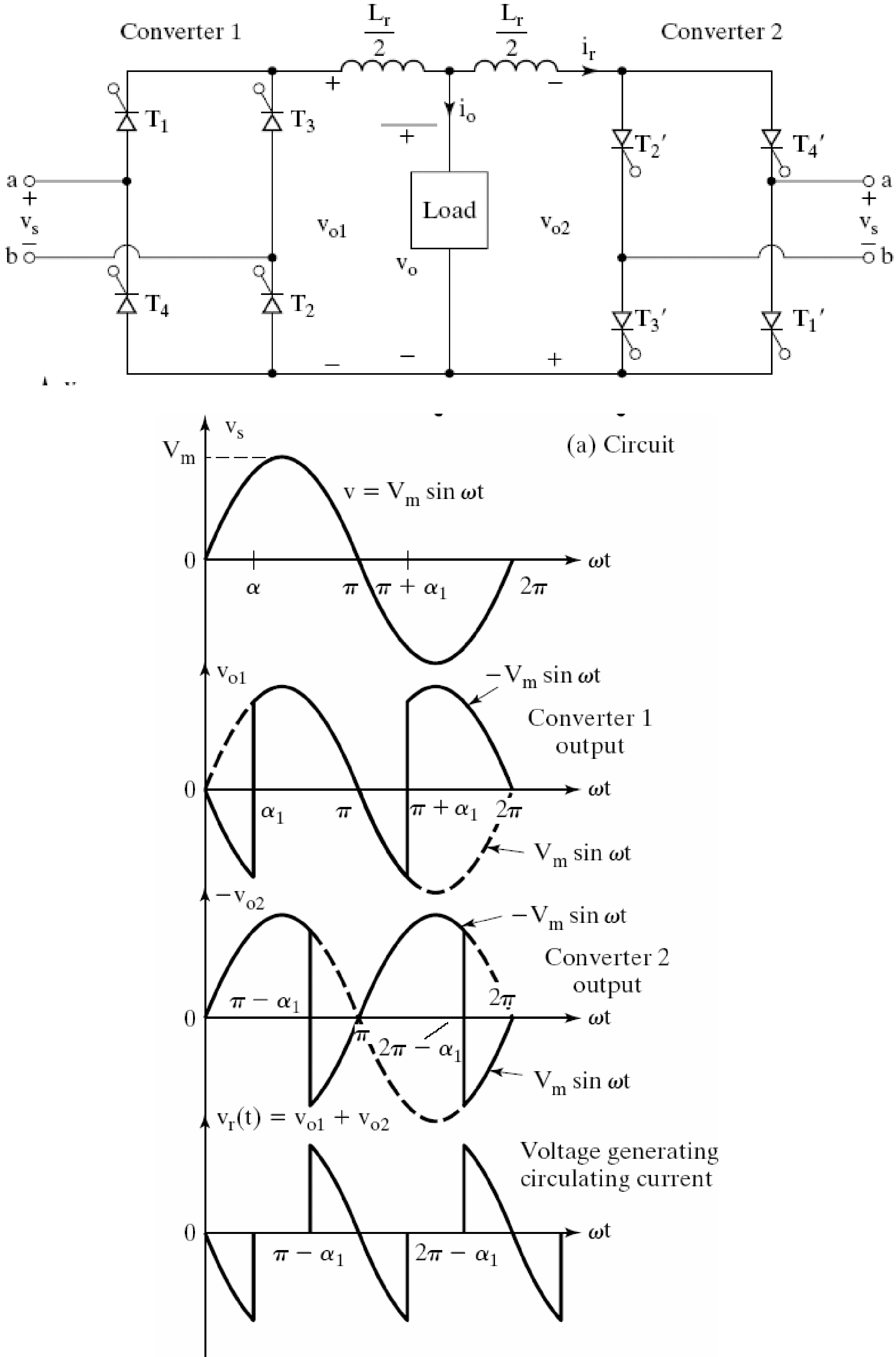
$$I_{T(RMS)} = \frac{I_{O(RMS)}}{\sqrt{2}}$$

The average thyristor current can be calculated as

$$I_{T(Avg)} = I_{O(dc)} \times \left(\frac{\pi}{2\pi}\right) = I_{O(dc)} \times \left(\frac{1}{2}\right)$$

$$I_{T(Avg)} = \frac{I_{O(dc)}}{2}$$

SINGLE PHASE DUAL CONVERTER



(a) Circuit

(b) Waveforms

We have seen in the case of a single phase full converter with inductive loads the converter can operate in two different quadrants in the V_{dc} versus I_{dc} operating diagram. If two single phase full converters are connected in parallel and in opposite direction (connected in back to back) across a common load four quadrant operation is possible. Such a converter is called as a dual converter which is shown in the figure.

The dual converter system will provide four quadrant operation and is normally used in high power industrial variable speed drives. The converter number 1 provides a positive dc output voltage and a positive dc load current, when operated in the rectification mode.

The converter number 2 provides a negative dc output voltage and a negative dc load current when operated in the rectification mode. We can thus have bi-directional load current and bi-directional dc output voltage. The magnitude of output dc load voltage and the dc load current can be controlled by varying the trigger angles α_1 & α_2 of the converters 1 and 2 respectively.

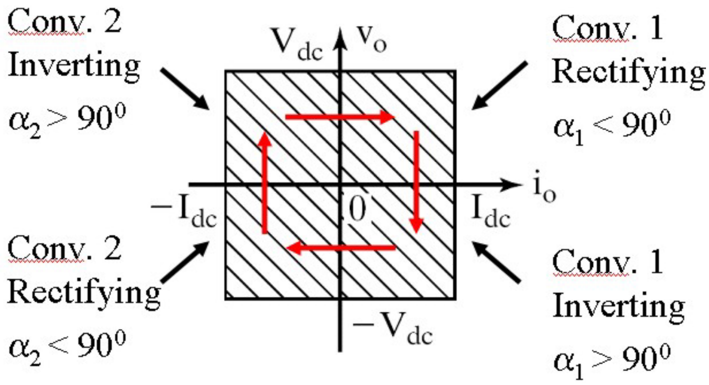


Fig.: Four quadrant operation of a dual converter

There are two modes of operations possible for a dual converter system.

- Non circulating current mode of operation (circulating current free mode of operation).
- Circulating current mode of operation.

NON CIRCULATING CURRENT MODE OF OPERATION (CIRCULATING CURRENT FREE MODE OF OPERATION)

In this mode of operation only one converter is switched on at a time while the second converter is switched off. When the converter 1 is switched on and the gate trigger signals are released to the gates of thyristors in converter 1, we get an average output voltage across the load, which can be varied by adjusting the trigger angle α_1 of the converter 1. If α_1 is less than 90^0, the converter 1 operates as a controlled rectifier and converts the input ac power into dc output power to feed the load. V_{dc} and I_{dc} are both positive and the operation occurs in the first quadrant. The average output power $P_{dc} = V_{dc} \times I_{dc}$ is positive. The power flows from the input ac supply to the load. When α_1 is increased above 90^0 converter 1 operates as a line commutated inverter and V_{dc} becomes negative while I_{dc} is positive and the output power P_{dc} becomes negative. The power is fed back from the load circuit to the input ac source through the converter 1. The load current falls to zero when the load energy is utilized completely.

The second converter 2 is switched on after a small delay of about 10 to 20 mill seconds to allow all the thyristors of converter 1 to turn off completely. The gate signals are released to the thyristor gates of converter 2 and the trigger angle α_2 is adjusted such that $0 \le \alpha_2 \le 90^0$ so that converter 2 operates as a controlled rectifier. The dc output voltage V_{dc} and I_{dc} are both negative and the load current flows in the reverse direction. The magnitude of V_{dc} and I_{dc} are controlled by the trigger angle α_2. The operation occurs in the third quadrant where V_{dc} and I_{dc} are both negative and output power P_{dc} is positive and the converter 2 operates as a controlled rectifier and converts the ac supply power into dc output power which is fed to the load.

When we want to reverse the load current flow so that I_{dc} is positive we have to operate converter 2 in the inverter mode by increasing the trigger angle α_2 above 90^0. When α_2 is made greater than 90^0, the converter 2 operates as a line commutated inverter and the load power (load energy) is fed back to ac mains. The current falls to zero when all the load energy is utilized and the converter 1 can be switched on after a short delay of 10 to 20 milli seconds to ensure that the converter 2 thyristors are completely turned off.

The advantage of non circulating current mode of operation is that there is no circulating current flowing between the two converters as only one converter operates and conducts at a time while the other converter is switched off. Hence there is no need of the series current limiting inductors between the outputs of the two converters. The current rating of thyristors is low in this mode.

But the disadvantage is that the load current tends to become discontinuous and the transfer characteristic becomes non linear. The control circuit becomes complex and the output response is sluggish as the load current reversal takes some time due to the time delay between the switching off of one converter and the switching on of the other converter. Hence the output dynamic response is poor. Whenever a fast and frequent reversal of the load current is required, the dual converter is operated in the circulating current mode.

CIRCULATING CURRENT MODE OF OPERATION

In this mode of operation both the converters 1 and 2 are switched on and operated simultaneously and both the converters are in a state of conduction. If converter 1 is operated as a controlled rectifier by adjusting the trigger angle α_1 between 0 to 90^0 the second converter 2 is operated as a line commutated inverter by increasing its trigger angle α_2 above 90^0. The trigger angles α_1 and α_2 are adjusted such that they produce the same average dc output voltage across the load terminals.

The average dc output voltage of converter 1 is

$$V_{dc1} = \frac{2V_m}{\pi}\cos\alpha_1$$

The average dc output voltage of converter 2 is

$$V_{dc2} = \frac{2V_m}{\pi}\cos\alpha_2$$

In the dual converter operation one converter is operated as a controlled rectifier with $\alpha_1 < 90^0$ and the second converter is operated as a line commutated inverter in the inversion mode with $\alpha_2 > 90^0$.

$$V_{dc1} = -V_{dc2}$$

$$\frac{2V_m}{\pi}\cos\alpha_1 = \frac{-2V_m}{\pi}\cos\alpha_2 = \frac{2V_m}{\pi}\left(-\cos\alpha_2\right)$$

Therefore $\quad \cos\alpha_1 = -\cos\alpha_2$ or $\cos\alpha_2 = -\cos\alpha_1 = \cos\left(\pi - \alpha_1\right)$

Therefore $\quad \alpha_2 = \left(\pi - \alpha_1\right)$ or $\left(\alpha_1 + \alpha_2\right) = \pi$ radians

Which gives $\quad \alpha_2 = \left(\pi - \alpha_1\right)$

When the trigger angle α_1 of converter 1 is set to some value the trigger angle α_2 of the second converter is adjusted such that $\alpha_2 = \left(180^0 - \alpha_1\right)$. Hence for circulating current mode of operation where both converters are conducting at the same time $\left(\alpha_1 + \alpha_2\right) = 180^0$ so that they produce the same dc output voltage across the load.

When $\alpha_1 < 90^0$ (say $\alpha_1 = 30^0$) the converter 1 operates as a controlled rectifier and converts the ac supply into dc output power and the average load current I_{dc} is positive. At the same time the converter 2 is switched on and operated as a line commutated inverter, by adjusting the trigger angle α_2 such that $\alpha_2 = \left(180^0 - \alpha_1\right)$, which is equal to 150^0, when $\alpha_1 = 30^0$. The converter 2 will operate in the inversion mode and feeds the load energy back to the ac supply. When we want to reverse the load current flow we have to switch the roles of the two converters.

When converter 2 is operated as a controlled rectifier by adjusting the trigger angle α_2 such that $\alpha_2 < 90^0$, the first converter1 is operated as a line commutated inverter, by adjusting the trigger angle α_1 such that $\alpha_1 > 90^0$. The trigger angle α_1 is adjusted such that $\alpha_1 = \left(180^0 - \alpha_2\right)$ for a set value of α_2.

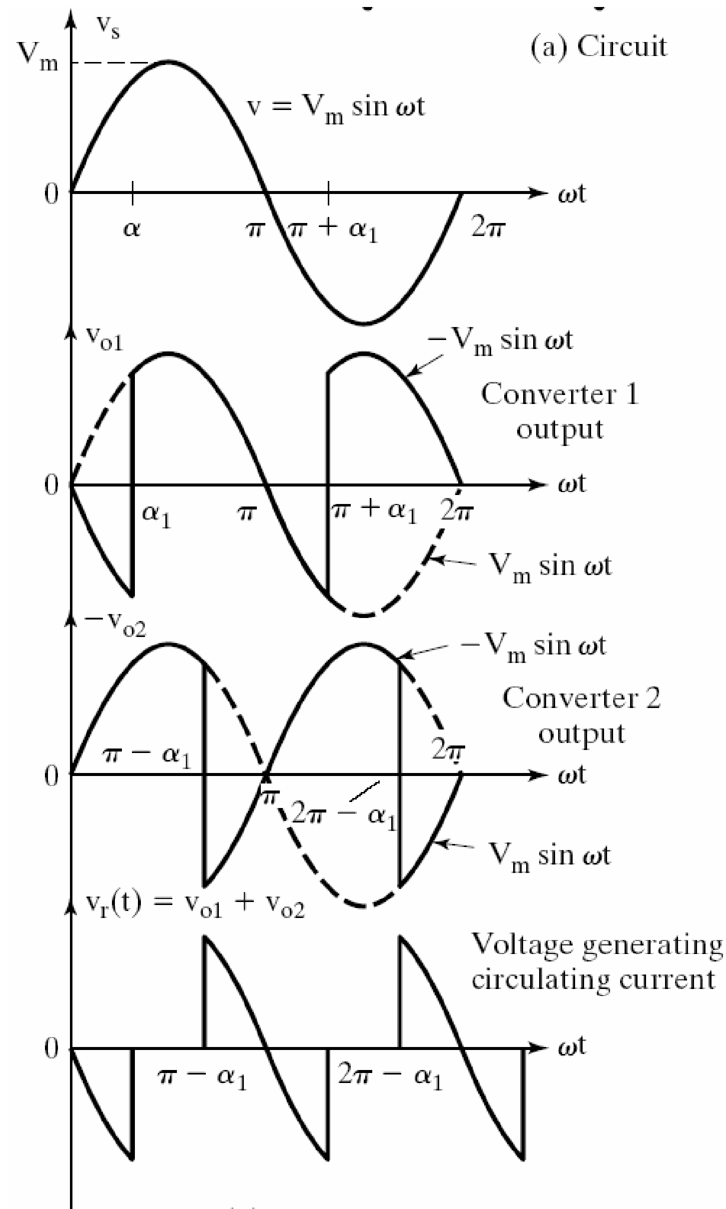
In the circulating current mode a current builds up between the two converters even when the load current falls to zero. In order to limit the circulating current flowing between the two converters, we have to include current limiting reactors in series between the output terminals of the two converters.

The advantage of the circulating current mode of operation is that we can have faster reversal of load current as the two converters are in a state of conduction simultaneously. This greatly improves the dynamic response of the output giving a faster dynamic response. The output voltage and the load current can be linearly varied by adjusting the trigger angles $\alpha_1 \& \alpha_2$ to obtain a smooth and linear output control. The control circuit becomes relatively simple. The transfer characteristic between the output voltage and the trigger angle is linear and hence the output response is very fast. The load current is free to flow in either direction at any time. The reversal of the load current can be done in a faster and smoother way.

The disadvantage of the circulating current mode of operation is that a current flows continuously in the dual converter circuit even at times when the load current is zero. Hence we should connect current limiting inductors (reactors) in order to limit the peak circulating current within specified value. The circulating current flowing through the series inductors gives rise to increased power losses, due to dc voltage drop across the series inductors which decreases the efficiency. Also the power factor of operation is low. The current limiting series inductors are heavier and bulkier which increases the cost and weight of the dual converter system.

The current flowing through the converter thyristors is much greater than the dc load current. Hence the thyristors should be rated for a peak thyristor current of $I_{T(\max)} = I_{dc(\max)} + i_{r(\max)}$, where $I_{dc(\max)}$ is the maximum dc load current and $i_{r(\max)}$ is the maximum value of the circulating current.

TO CALCULATE THE CIRCULATING CURRENT



(b) Waveforms

Fig.: Waveforms of dual converter

As the instantaneous output voltages of the two converters are out of phase, there will be an instantaneous voltage difference and this will result in circulating current between the two converters. In order to limit the circulating current, current limiting reactors are connected in series between the outputs of the two converters. This circulating current will not flow through the load and is normally limited by the current reactor L_r .

If v_{O1} and v_{O2} are the instantaneous output voltages of the converters 1 and 2, respectively the circulating current can be determined by integrating the instantaneous voltage difference (which is the voltage drop across the circulating current reactor L_r), starting from $\omega t = (2\pi - \alpha_1)$. As the two average output voltages during the interval $\omega t = (\pi + \alpha_1)$ to $(2\pi - \alpha_1)$ are equal and opposite their contribution to the instantaneous circulating current i_r is zero.

$$i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} v_r.d(\omega t)\right]; \quad v_r = (v_{O1} - v_{O2})$$

As the output voltage v_{O2} is negative

$$v_r = (v_{O1} + v_{O2})$$

Therefore $$i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} (v_{O1} + v_{O2}).d(\omega t)\right];$$

$$v_{O1} = -V_m \sin\omega t \text{ for } (2\pi - \alpha_1) \text{ to } \omega t$$

$$i_r = \frac{V_m}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} -\sin\omega t.d(\omega t) - \int_{(2\pi-\alpha_1)}^{\omega t} \sin\omega t.d(\omega t)\right]$$

$$i_r = \frac{V_m}{\omega L_r}\left[(\cos\omega t)\Big/_{(2\pi-\alpha_1)}^{\omega t} + (\cos\omega t)\Big/_{(2\pi-\alpha_1)}^{\omega t}\right]$$

$$i_r = \frac{V_m}{\omega L_r}\left[(\cos\omega t) - \cos(2\pi - \alpha_1) + (\cos\omega t) - \cos(2\pi - \alpha_1)\right]$$

$$i_r = \frac{V_m}{\omega L_r}\left[2\cos\omega t - 2\cos(2\pi - \alpha_1)\right]$$

$$i_r = \frac{2V_m}{\omega L_r}(\cos\omega t - \cos\alpha_1)$$

The instantaneous value of the circulating current depends on the delay angle.

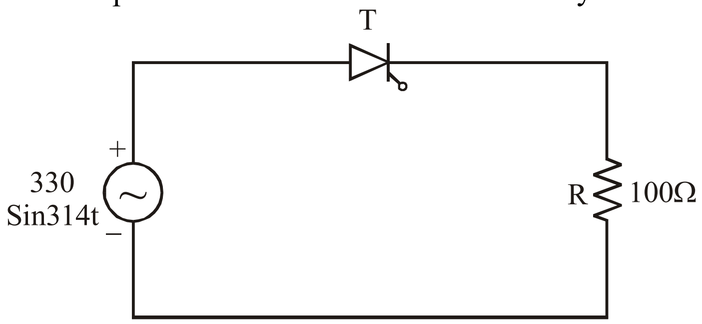
For trigger angle (delay angle) $\alpha_1 = 0$, its magnitude becomes minimum when $\omega t = n\pi,\ n = 0, 2, 4, ...$ and magnitude becomes maximum when $\omega t = n\pi,\ n = 1, 3, 5, ...$

If the peak load current is I_p, one of the converters that controls the power flow may carry a peak current of $I_p + \frac{4V_m}{\omega L_r}$,

Where $$I_p = I_{L(\max)} = \frac{V_m}{R_L}, \ \& \ i_{r(\max)} = \frac{4V_m}{\omega L_r}$$

Problems

1. What will be the average power in the load for the circuit shown, when $\alpha = \frac{\pi}{4}$. Assume SCR to be ideal. Supply voltage is 330 sin314t. Also calculate the RMS power and the rectification efficiency.



The circuit is that of a single phase half wave controlled rectifier with a resistive load

$$V_{dc} = \frac{V_m}{2\pi}(1+\cos\alpha) \quad ; \qquad \alpha = \frac{\pi}{4} \ radians$$

$$V_{dc} = \frac{330}{2\pi}\left(1+\cos\left(\frac{\pi}{4}\right)\right)$$

$$V_{dc} = 89.66 \text{ Volts}$$

Average Power $= \frac{V_{dc}^2}{R} = \frac{89.66^2}{100} = 80.38$ Watts

$$I_{dc} = \frac{V_{dc}}{R} = \frac{89.66}{100} = 0.8966 \text{ Amps}$$

$$V_{RMS} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_{RMS} = \frac{330}{2}\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{4} + \frac{\sin 2\times\frac{\pi}{4}}{2}\right)\right]^{\frac{1}{2}}$$

$$V_{RMS} = 157.32 \ V$$

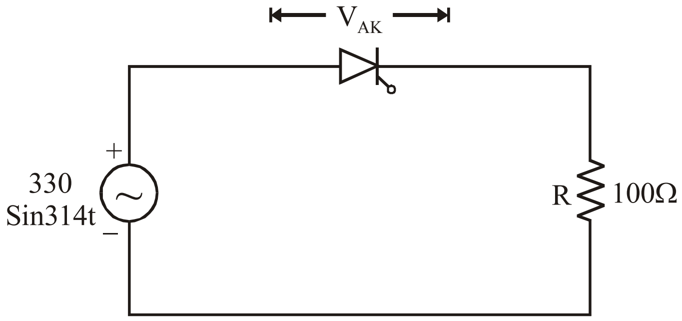
RMS Power (AC power)

$$= \frac{V_{RMS}^2}{R} = \frac{157.32^2}{100} = 247.50 \text{ Watts}$$

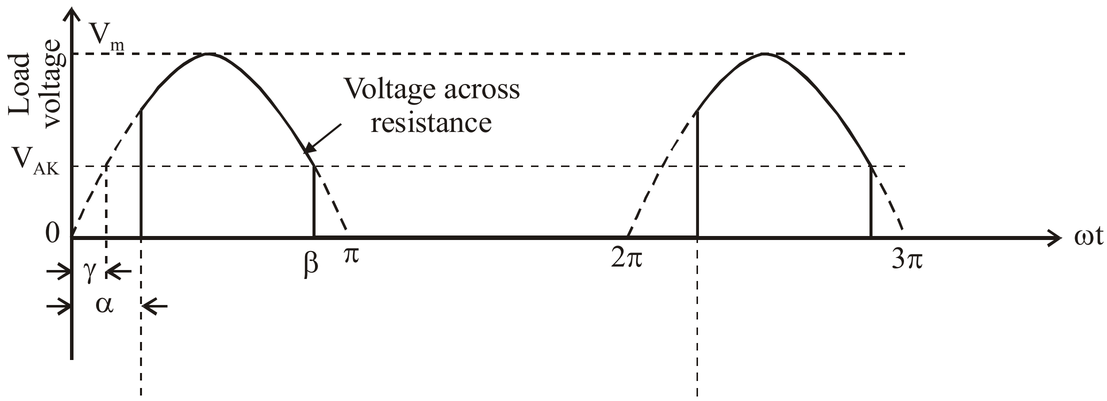
$$\text{Rectification Efficiency} = \frac{\text{Average power}}{\text{RMS power}}$$

$$= \frac{80.38}{247.47} = 0.3248$$

2. In the circuit shown find out the average voltage across the load assuming that the conduction drop across the SCR is 1 volt. Take $\alpha = 45^0$.



The wave form of the load voltage is shown below (not to scale).



It is observed that the SCR turns off when $\omega t = \beta$, where $\beta = (\pi - \gamma)$ because the SCR turns-off for anode supply voltage below 1 Volt.

$$V_{AK} = V_m \sin\gamma = 1 \text{ volt (given)}$$

Therefore $\gamma = \sin^{-1}\left(\frac{V_{AK}}{V_m}\right) = \sin^{-1}\left(\frac{1}{330}\right) = 0.17^0 \left(0.003 \text{ radians}\right)$

$$\beta = \left(180^0 - \gamma\right) \quad ; \text{ By symmetry of the curve.}$$

$$\beta = 179.83^0 \quad ; \ 3.138 \text{ radians.}$$

$$V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\beta}\left(V_m \sin\omega t - V_{AK}\right) d\left(\omega t\right)$$

$$V_{dc} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t.d(\omega t) - V_{AK}\int_{\alpha}^{\beta} d(\omega t)\right]$$

$$V_{dc} = \frac{1}{2\pi}\left[V_m(-\cos\omega t)\Big/_{\alpha}^{\beta} - V_{AK}(\omega t)\Big/_{\alpha}^{\beta}\right]$$

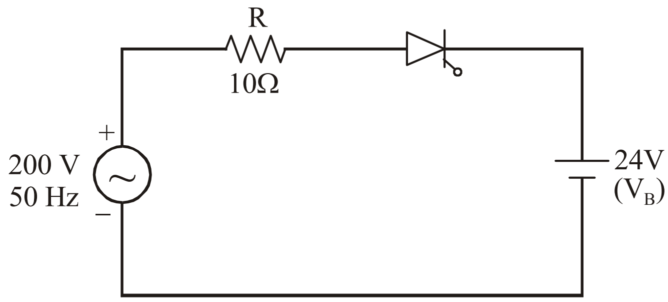
$$V_{dc} = \frac{1}{2\pi}\left[V_m(\cos\alpha - \cos\beta) - V_{AK}(\beta - \alpha)\right]$$

$$V_{dc} = \frac{1}{2\pi}\left[330\left(\cos 45^0 - \cos 179.83^0\right) - 1\left(3.138 - 0.003\right)\right]$$

$$V_{dc} = 89.15 \text{ Volts}$$

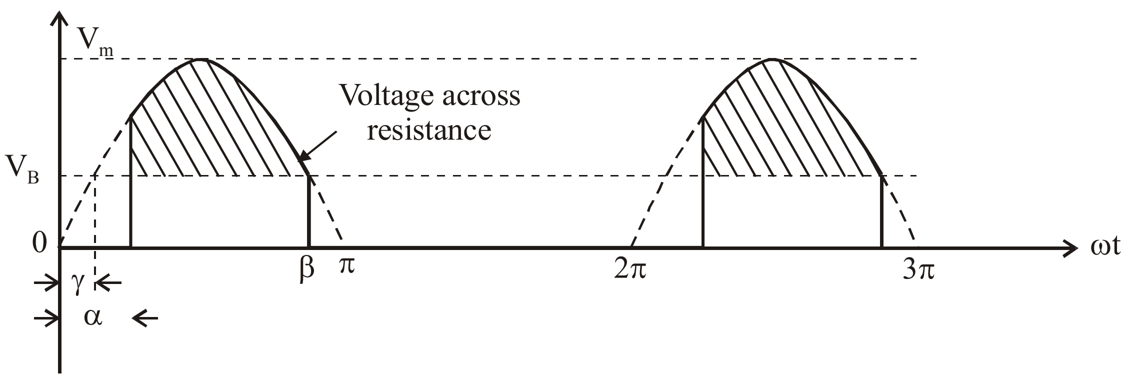
Note: β and α values should be in radians

3. In the figure find out the battery charging current when $\alpha = \frac{\pi}{4}$. Assume ideal SCR.



Solution

It is obvious that the SCR cannot conduct when the instantaneous value of the supply voltage is less than 24 V, the battery voltage. The load voltage waveform is as shown (voltage across ion).



$$V_B = V_m \sin\gamma$$

$$24 = 200\sqrt{2}\sin\gamma$$

$$\gamma = \sin^{-1}\left(\frac{24}{200\times\sqrt{2}}\right) = 4.8675^0 = 0.085 \text{ radians}$$

$$\beta = \pi - \gamma = 3.056 \text{ radians}$$

Average value of voltage across 10Ω

$$= \frac{1}{2\pi}\left[\int_{\alpha}^{\beta}\left(V_m \sin\omega t - V_B\right).d\left(\omega t\right)\right]$$

(The integral gives the shaded area)

$$= \frac{1}{2\pi}\left[\int_{\frac{\pi}{4}}^{3.056}\left(200\times\sqrt{2}\sin\omega t - 24\right).d\left(\omega t\right)\right]$$

$$= \frac{1}{2\pi}\left[200\sqrt{2}\left(\cos\frac{\pi}{4} - \cos 3.056\right) - 24\left(3.056 - \frac{\pi}{4}\right)\right]$$

$$= 68 \text{ Vots}$$

Therefore charging current

$$= \frac{\text{Average voltage across R}}{R}$$

$$= \frac{68}{10} = 6.8 \text{ Amps}$$

Note: If value of γ is more than α, then the SCR will trigger only at $\omega t = \gamma$, (assuming that the gate signal persists till then), when it becomes forward biased.

Therefore $$V_{dc} = \frac{1}{2\pi}\left[\int_{\gamma}^{\beta}\left(V_m \sin\omega t - V_B\right).d\left(\omega t\right)\right]$$

4. In a single phase full wave rectifier supply is 200 V AC. The load resistance is 10Ω, $\alpha = 60^0$. Find the average voltage across the load and the power consumed in the load.

Solution

In a single phase full wave rectifier

$$V_{dc} = \frac{V_m}{\pi}\left(1 + \cos\alpha\right)$$

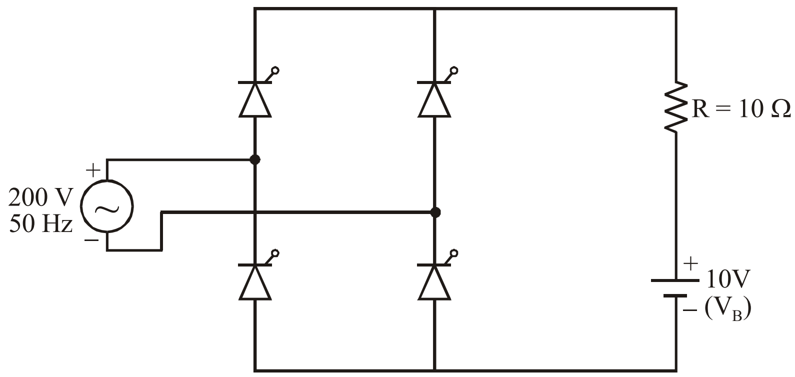
$$V_{dc} = \frac{200 \times \sqrt{2}}{\pi}\left(1 + \cos 60^0\right)$$

$$V_{dc} = 135 \text{ Volts}$$

Average Power

$$= \frac{V_{dc}^2}{R} = \frac{135^2}{10} = 1.823\ kW$$

5. In the circuit shown find the charging current if the trigger angle $\alpha = 90^0$.



Solution
With the usual notation

$$V_B = V_m \sin \gamma$$

$$10 = 200\sqrt{2} \sin \gamma$$

Therefore $\quad \gamma = \sin^{-1}\left(\frac{10}{200 \times \sqrt{2}}\right) = 0.035 \text{ radians}$

$$\alpha = 90^0 = \frac{\pi}{2} \text{ radians} \quad ; \ \beta = (\pi - \gamma) = 3.10659$$

Average voltage across $10\Omega = \frac{2}{2\pi}\left[\int_{\alpha}^{\beta}\left(V_m \sin \omega t - V_B\right).d(\omega t)\right]$

$$= \frac{1}{\pi}\left[-V_m \cos \omega t - V_B(\omega t)\right]_{\alpha}^{\beta}$$

$$= \frac{1}{\pi}\left[V_m(\cos \alpha - \cos \beta) - V_B(\beta - \alpha)\right]$$

$$= \frac{1}{\pi}\left[200\times\sqrt{2}\left(\cos\frac{\pi}{2} - \cos 3.106\right) - 10\left(3.106 - \frac{\pi}{2}\right)\right]$$

$$= 85 \text{ V}$$

Note that the values of α & β are in radians.

$$\text{Charging current} = \frac{\text{dc voltage across resistance}}{\text{resistance}}$$

$$= \frac{85}{10} = 8.5 \text{ Amps}$$

6. A single phase full wave controlled rectifier is used to supply a resistive load of 10 Ω from a 230 V, 50 Hz, supply and firing angle of 90^0. What is its mean load voltage? If a large inductance is added in series with the load resistance, what will be the new output load voltage?

Solution

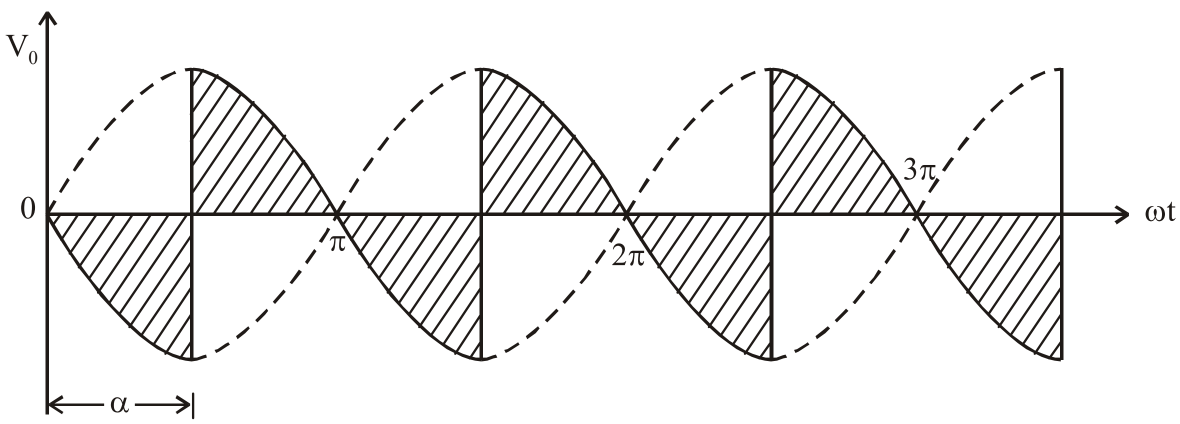
For a single phase full wave controlled rectifier with resistive load,

$$V_{dc} = \frac{V_m}{\pi}\left(1+\cos\alpha\right)$$

$$V_{dc} = \frac{230\times\sqrt{2}}{\pi}\left(1+\cos\frac{\pi}{2}\right)$$

$$V_{dc} = 103.5 \text{ Volts}$$

When a large inductance is added in series with the load, the output voltage wave form will be as shown below, for trigger angle $\alpha = 90^0$.



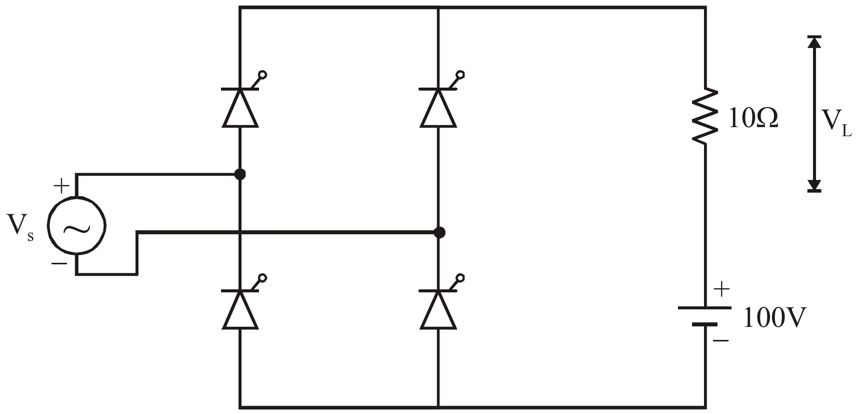
$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Since $\alpha = \frac{\pi}{2}$; $cos\alpha = \cos\left(\frac{\pi}{2}\right) = 0$

Therefore $V_{dc} = 0$ and this is evident from the waveform also.

7. The figure shows a battery charging circuit using SCRs. The input voltage to the circuit is 230 V RMS. Find the charging current for a firing angle of 45^0. If any one of the SCR is open circuited, what is the charging current?

Solution



With the usual notations

$$V_S = V_m \sin\omega t$$

$$V_S = \sqrt{2} \times 230 \sin\omega t$$

$V_m \sin\gamma = V_B$, the battery voltage

$$\sqrt{2} \times 230 \sin\gamma = 100$$

Therefore $\gamma = \sin^{-1}\left(\frac{100}{\sqrt{2} \times 230}\right)$

$\gamma = 17.9^0$ or 0.312 radians

$$\beta = (\pi - \gamma) = (\pi - 0.312)$$

$\beta = 2.829$ radians

Average value of voltage across load resistance

$$= \frac{2}{2\pi}\left[\int_{\alpha}^{\beta}\left(V_m \sin\omega t - V_B\right) d\left(\omega t\right)\right]$$

$$= \frac{1}{\pi}\left[-V_m \cos\omega t - V_B\left(\omega t\right)\right]_{\alpha}^{\beta}$$

$$= \frac{1}{\pi}\left[V_m\left(\cos\alpha - \cos\beta\right) - V_B\left(\beta - \alpha\right)\right]$$

$$= \frac{1}{\pi}\left[230\times\sqrt{2}\left(\cos\frac{\pi}{4} - \cos 2.829\right) - 100\left(2.829 - \frac{\pi}{4}\right)\right]$$

$$= \frac{1}{\pi}\left[230\times\sqrt{2}\left(0.707 + 0.9517\right) - 204.36\right]$$

$$= 106.68 \text{ Volts}$$

$$\text{Charging current } = \frac{\text{Voltage across resistance}}{R}$$

$$= \frac{106.68}{10} = 10.668 \text{ Amps}$$

If one of the SCRs is open circuited, the circuit behaves like a half wave rectifier. The average voltage across the resistance and the charging current will be half of that of a full wave rectifier.

$$\text{Therefore Charging Current } = \frac{10.668}{2} = 5.334 \text{ Amps}$$

THREE PHASE CONTROLLED RECTIFIERS

INTRODUCTION TO 3-PHASE CONTROLLED RECTIFIERS

Single phase half controlled bridge converters & fully controlled bridge converters are used extensively in industrial applications up to about 15kW of output power. The single phase controlled rectifiers provide a maximum dc output of $V_{dc(\max)} = \frac{2V_m}{\pi}$.

The output ripple frequency is equal to the twice the ac supply frequency. The single phase full wave controlled rectifiers provide two output pulses during every input supply cycle and hence are referred to as two pulse converters.

Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load.

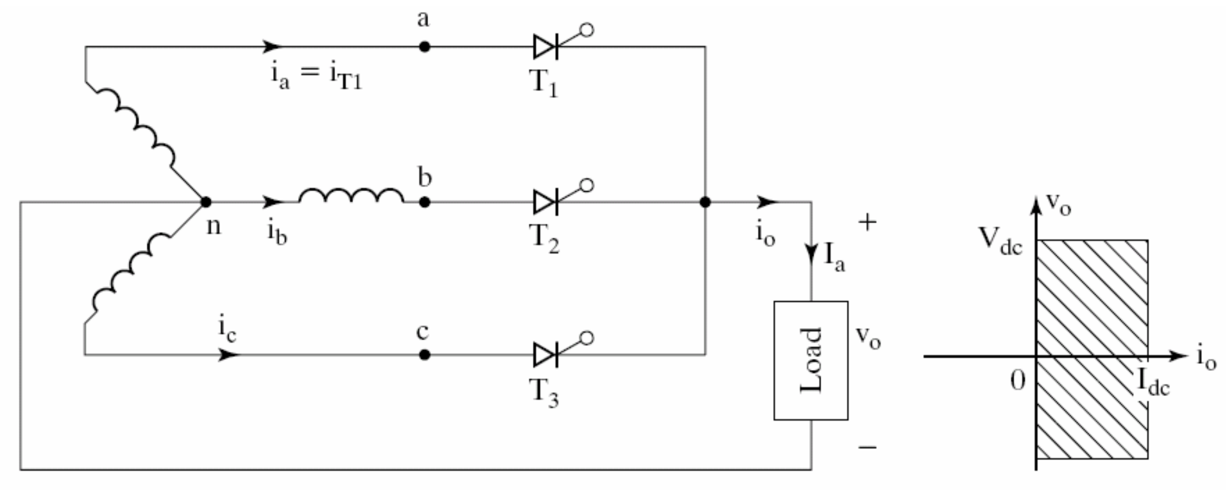
Features of 3-phase controlled rectifiers are

- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage and higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current

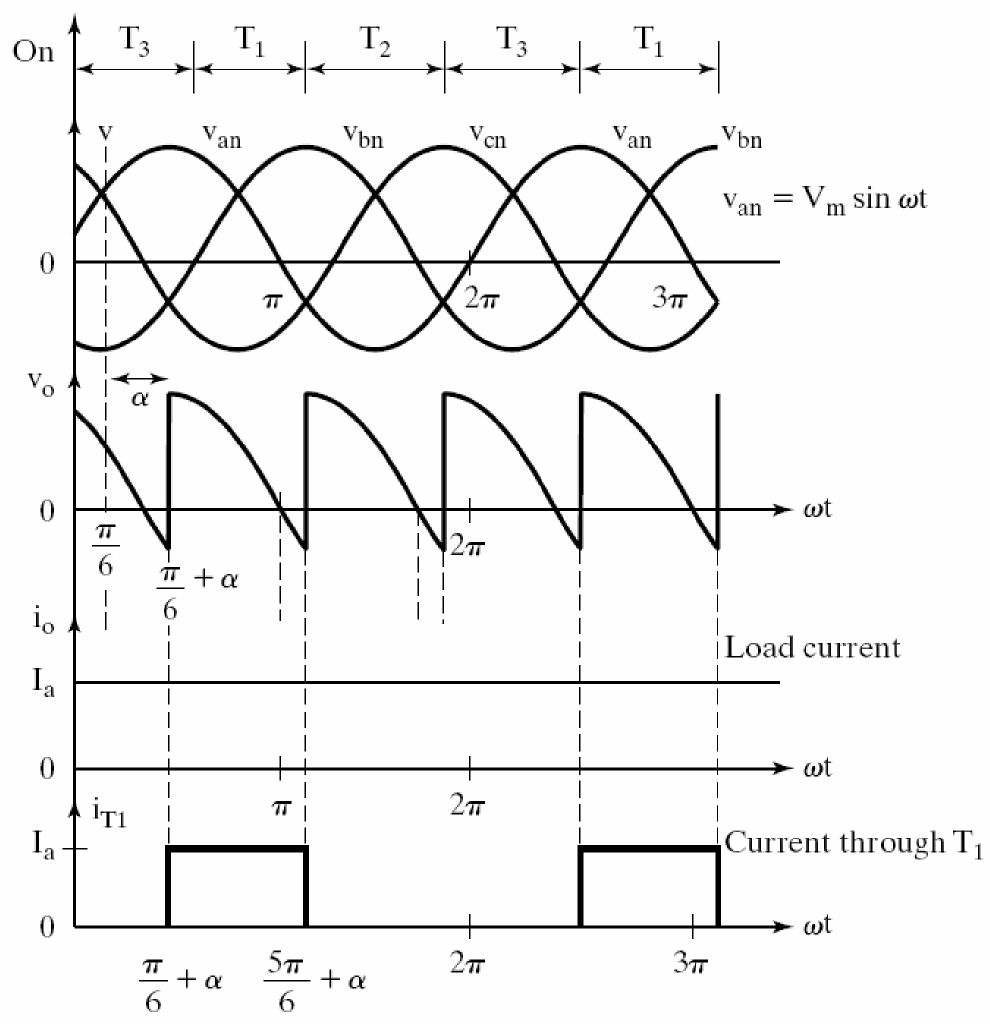
Three phase controlled rectifiers are extensively used in high power variable speed industrial dc drives.

3-PHASE HALF WAVE CONVERTER

Three single phase half-wave converters are connected together to form a three phase half-wave converter as shown in the figure.



(a) Circuit (b) Quadrant



(c) For inductive load

THEE PHASE SUPPLY VOLTAGE EQUATIONS

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t; \quad V_m = \text{ Max. Phase Voltage}$$

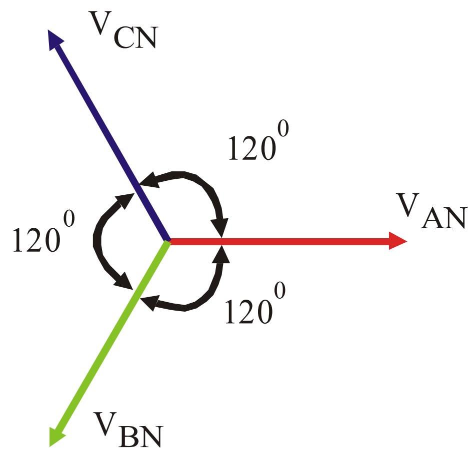
$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t - 240^0\right)$$



Vector diagram of 3-phase supply voltages

The 3-phase half wave converter combines three single phase half wave controlled rectifiers in one single circuit feeding a common load. The thyristor T_1 in series with one of the supply phase windings $'a-n'$ acts as one half wave controlled rectifier. The second thyristor T_2 in series with the supply phase winding $'b-n'$ acts as the second half wave controlled rectifier. The third thyristor T_3 in series with the supply phase winding $'c-n'$ acts as the third half wave controlled rectifier.

The 3-phase input supply is applied through the star connected supply transformer as shown in the figure. The common neutral point of the supply is connected to one end of the load while the other end of the load connected to the common cathode point.

When the thyristor T_1 is triggered at $\omega t = \left(\frac{\pi}{6}+\alpha\right) = \left(30^0+\alpha\right)$, the phase voltage v_{an} appears across the load when T_1 conducts. The load current flows through the supply phase winding $'a-n'$ and through thyristor T_1 as long as T_1 conducts.

When thyristor T_2 is triggered at $\omega t = \left(\frac{5\pi}{6}+\alpha\right) = \left(150^0+\alpha\right)$, T_1 becomes reverse biased and turns-off. The load current flows through the thyristor T_2 and through the supply phase winding $'b-n'$. When T_2 conducts the phase voltage v_{bn} appears across the load until the thyristor T_3 is triggered .

When the thyristor T_3 is triggered at $\omega t = \left(\frac{3\pi}{2}+\alpha\right) = \left(270^0+\alpha\right)$, T_2 is reversed biased and hence T_2 turns-off. The phase voltage v_{cn} appears across the load when T_3 conducts.

When T_1 is triggered again at the beginning of the next input cycle the thyristor T_3 turns off as it is reverse biased naturally as soon as T_1 is triggered. The figure shows the 3-phase input supply voltages, the output voltage which appears across the load, and the load current assuming a constant and ripple free load current for a highly inductive load and the current through the thyristor T_1.

For a purely resistive load where the load inductance 'L = 0' and the trigger angle $\alpha > \left(\frac{\pi}{6}\right)$, the load current appears as discontinuous load current and each thyristor is naturally commutated when the polarity of the corresponding phase supply voltage reverses. The frequency of output ripple frequency for a 3-phase half wave converter is $3f_S$, where f_S is the input supply frequency.

The 3-phase half wave converter is not normally used in practical converter systems because of the disadvantage that the supply current waveforms contain dc components (i.e., the supply current waveforms have an average or dc value).

TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF A 3-PHASE HALF WAVE CONVERTER FOR CONTINUOUS LOAD CURRENT

The reference phase voltage is $v_{RN} = v_{an} = V_m \sin\omega t$. The trigger angle α is measured from the cross over points of the 3-phase supply voltage waveforms. When the phase supply voltage v_{an} begins its positive half cycle at $\omega t = 0$, the first cross over point appears at $\omega t = \left(\frac{\pi}{6}\right)$ $radians = 30^0$.

The trigger angle α for the thyristor T_1 is measured from the cross over point at $\omega t = 30^0$. The thyristor T_1 is forward biased during the period $\omega t = 30^0$ to 150^0, when the phase supply voltage v_{an} has a higher amplitude than the other phase supply voltages. Hence T_1 can be triggered between 30^0 to 150^0. When the thyristor T_1 is triggered at a trigger angle α, the average or dc output voltage for continuous load current is calculated using the equation

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} v_O.d(\omega t)\right]$$

Output voltage $\quad v_O = v_{an} = V_m \sin\omega t$ for $\omega t = (30^0 + \alpha)$ to $(150^0 + \alpha)$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\omega t.d(\omega t)\right]$$

As the output load voltage waveform has three output pulses during the input cycle of 2π radians

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \sin\omega t.d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\left(-\cos\omega t\right)\Big/_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

Note from the trigonometric relationship

$$\cos(A+B) = (\cos A.\cos B - \sin A.\sin B)$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}\right)\cos(\alpha)+\sin\left(\frac{5\pi}{6}\right)\sin(\alpha)+\cos\left(\frac{\pi}{6}\right).\cos(\alpha)-\sin\left(\frac{\pi}{6}\right)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(150^0\right)\cos(\alpha)+\sin\left(150^0\right)\sin(\alpha)+\cos\left(30^0\right).\cos(\alpha)-\sin\left(30^0\right)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(180^0-30^0\right)\cos(\alpha)+\sin\left(180^0-30^0\right)\sin(\alpha)+\cos\left(30^0\right).\cos(\alpha)-\sin\left(30^0\right)\sin(\alpha)\right]$$

Note: $\cos\left(180^0-30^0\right) = -\cos\left(30^0\right)$

$\sin\left(180^0-30^0\right) = \sin\left(30^0\right)$

Therefore

$$V_{dc} = \frac{3V_m}{2\pi}\left[+\cos\left(30^0\right)\cos(\alpha)+\sin\left(30^0\right)\sin(\alpha)+\cos\left(30^0\right).\cos(\alpha)-\sin\left(30^0\right)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\cos\left(30^0\right)\cos(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\times\frac{\sqrt{3}}{2}\cos(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\sqrt{3}\cos(\alpha)\right] = \frac{3\sqrt{3}V_m}{2\pi}\cos(\alpha)$$

$$V_{dc} = \frac{3V_{Lm}}{2\pi}\cos(\alpha)$$

Where

$V_{Lm} = \sqrt{3}V_m =$ Max. line to line supply voltage for a 3-phase star connected transformer.

The maximum average or dc output voltage is obtained at a delay angle $\alpha = 0$ and is given by

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}\ V_m}{2\pi}$$

Where

V_m is the peak phase voltage.

And the normalized average output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF THE OUTPUT VOLTAGE OF A 3-PHASE HALF WAVE CONVERTER FOR CONTINUOUS LOAD CURRENT

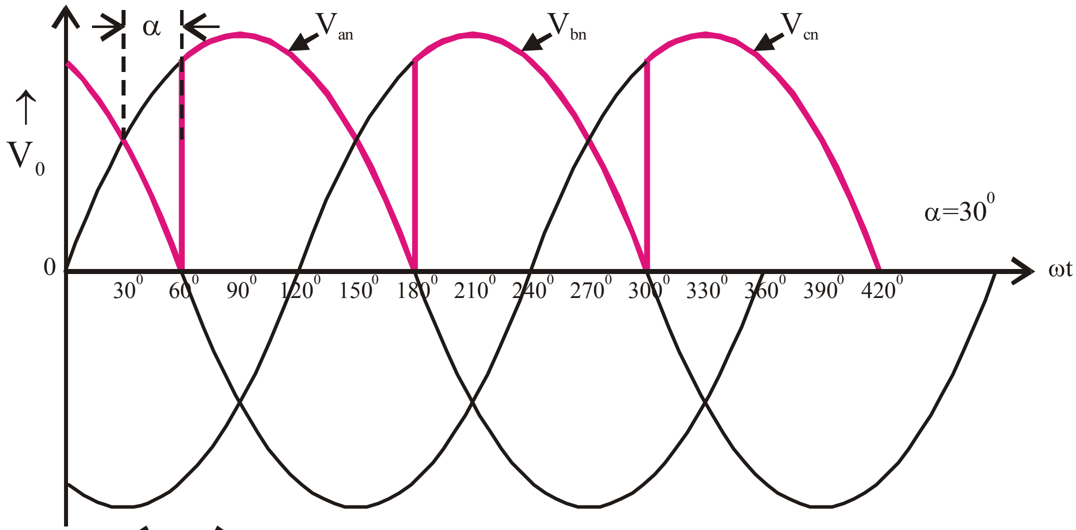
The rms value of output voltage is found by using the equation

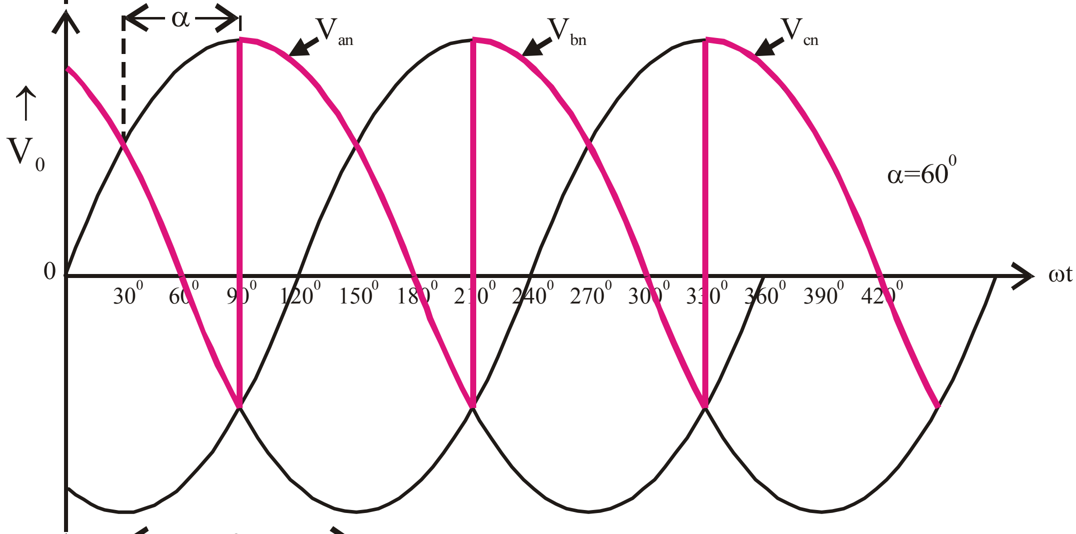
$$V_{O(RMS)} = \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t.d(\omega t)\right]^{\frac{1}{2}}$$

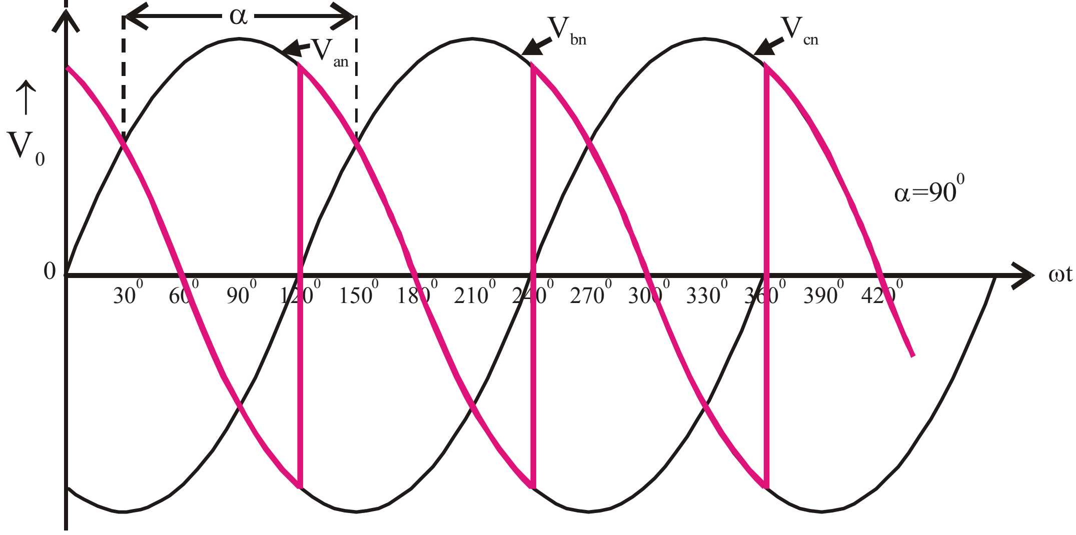
and we obtain

$$V_{O(RMS)} = \sqrt{3}V_m\left[\frac{1}{6}+\frac{\sqrt{3}}{8\pi}\cos 2\alpha\right]^{\frac{1}{2}}$$

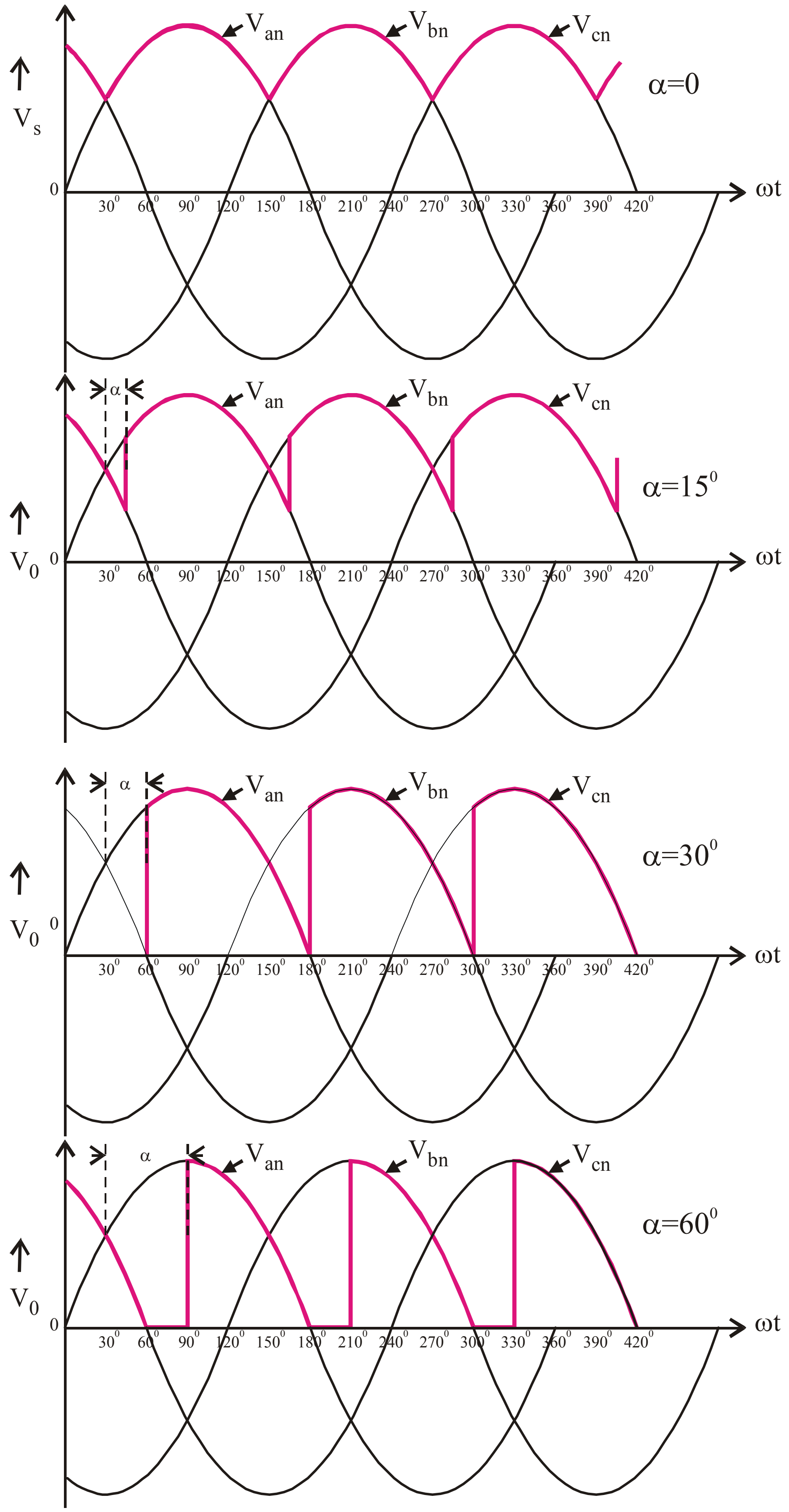
3 PHASE HALF WAVE CONTROLLED RECTIFIER OUTPUT VOLTAGE WAVEFORMS FOR DIFFERENT TRIGGER ANGLES WITH RL LOAD







3 PHASE HALF WAVE CONTROLLED RECTIFIER OUTPUT VOLTAGE WAVEFORMS FOR DIFFERENT TRIGGER ANGLES WITH R LOAD



TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF A 3 PHASE HALF WAVE CONVERTER WITH RESISTIVE LOAD OR RL LOAD WITH FWD.

In the case of a three-phase half wave controlled rectifier with resistive load, the thyristor T_1 is triggered at $\omega t = \left(30^0 + \alpha\right)$ and T_1 conducts up to $\omega t = 180^0 = \pi$ radians. When the phase supply voltage v_{an} decreases to zero at $\omega t = \pi$, the load current falls to zero and the thyristor T_1 turns off. Thus T_1 conducts from $\omega t = \left(30^0 + \alpha\right)$ to $\left(180^0\right)$.

Hence the average dc output voltage for a 3-pulse converter (3-phase half wave controlled rectifier) is calculated by using the equation

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\alpha+30^0}^{180^0} v_O.d\left(\omega t\right)\right]$$

$$v_O = v_{an} = V_m \sin\omega t;\ \text{for } \omega t = \left(\alpha + 30^0\right) \text{ to } \left(180^0\right)$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\alpha+30^0}^{180^0} V_m \sin\omega t.d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\alpha+30^0}^{180^0} \sin\omega t.d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha+30^0}^{180^0}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos 180^0 + \cos\left(\alpha + 30^0\right)\right]$$

Since $\cos 180^0 = -1$,

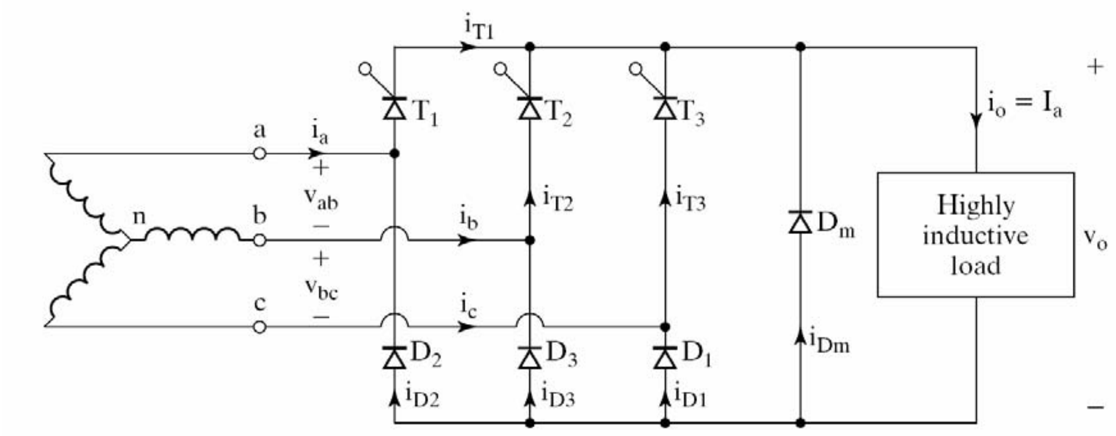
We get $$V_{dc} = \frac{3V_m}{2\pi}\left[1 + \cos\left(\alpha + 30^0\right)\right]$$

THREE PHASE SEMICONVERTERS

3-phase semi-converters are three phase half controlled bridge controlled rectifiers which employ three thyristors and three diodes connected in the form of a bridge configuration. Three thyristors are controlled switches which are turned on at appropriate times by applying appropriate gating signals. The three diodes conduct when they are forward biased by the corresponding phase supply voltages.

3-phase semi-converters are used in industrial power applications up to about 120kW output power level, where single quadrant operation is required. The power factor of 3-phase semi-converter decreases as the trigger angle α increases. The power factor of a 3-phase semi-converter is better than three phase half wave converter.

The figure shows a 3-phase semi-converter with a highly inductive load and the load current is assumed to be a constant and continuous load current with negligible ripple.



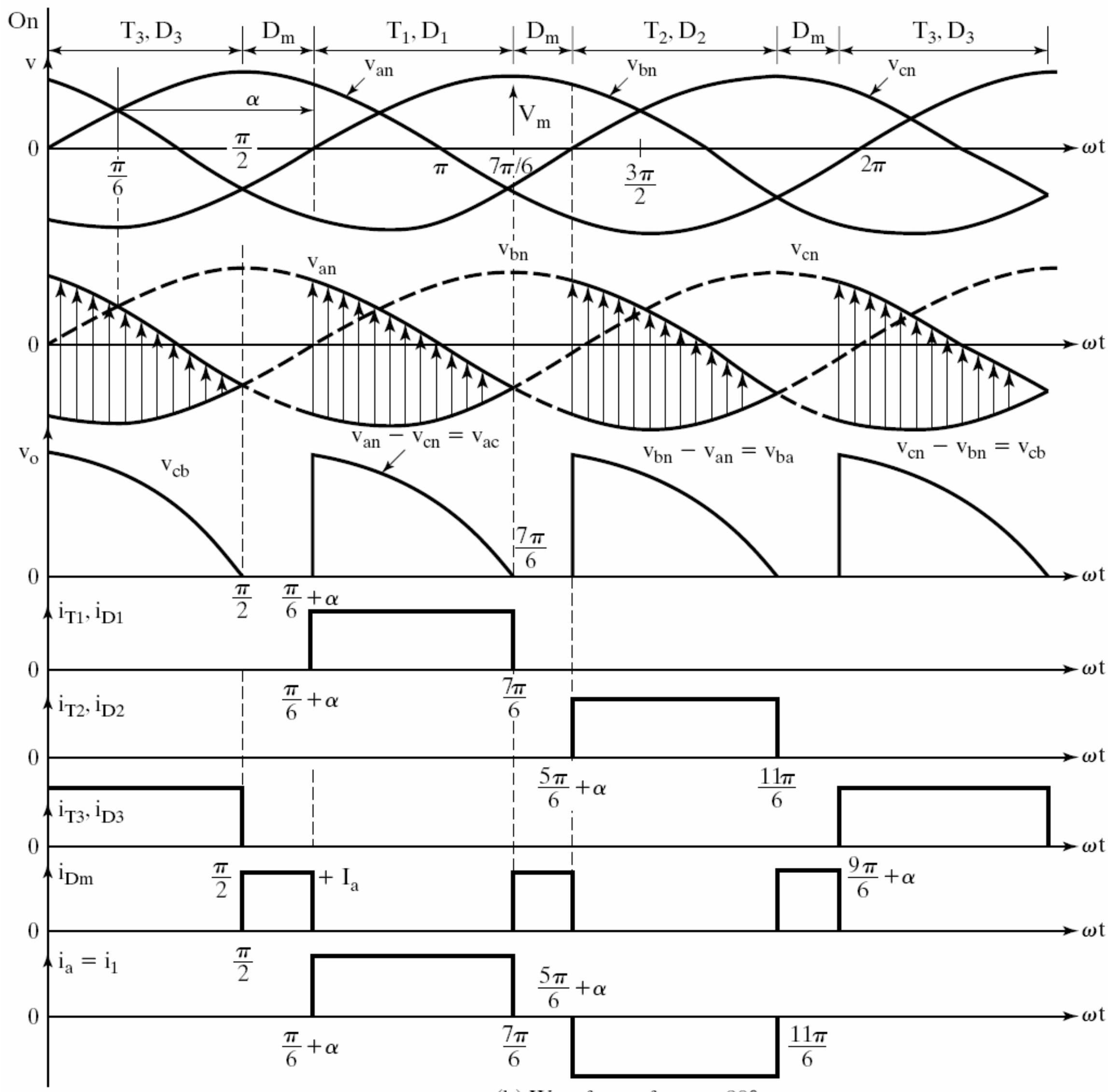
Thyristor T_1 is forward biased when the phase supply voltage v_{an} is positive and greater than the other phase voltages v_{bn} and v_{cn} . The diode D_1 is forward biased when the phase supply voltage v_{cn} is more negative than the other phase supply voltages.

Thyristor T_2 is forward biased when the phase supply voltage v_{bn} is positive and greater than the other phase voltages. Diode D_2 is forward biased when the phase supply voltage v_{an} is more negative than the other phase supply voltages.

Thyristor T_3 is forward biased when the phase supply voltage v_{cn} is positive and greater than the other phase voltages. Diode D_3 is forward biased when the phase supply voltage v_{bn} is more negative than the other phase supply voltages.

The figure shows the waveforms for the three phase input supply voltages, the output voltage, the thyristor and diode current waveforms, the current through the free wheeling diode D_m and the supply current i_a . The frequency of the output supply waveform is $3f_S$, where f_S is the input ac supply frequency. The trigger angle α can be varied from 0^0 to 180^0 .

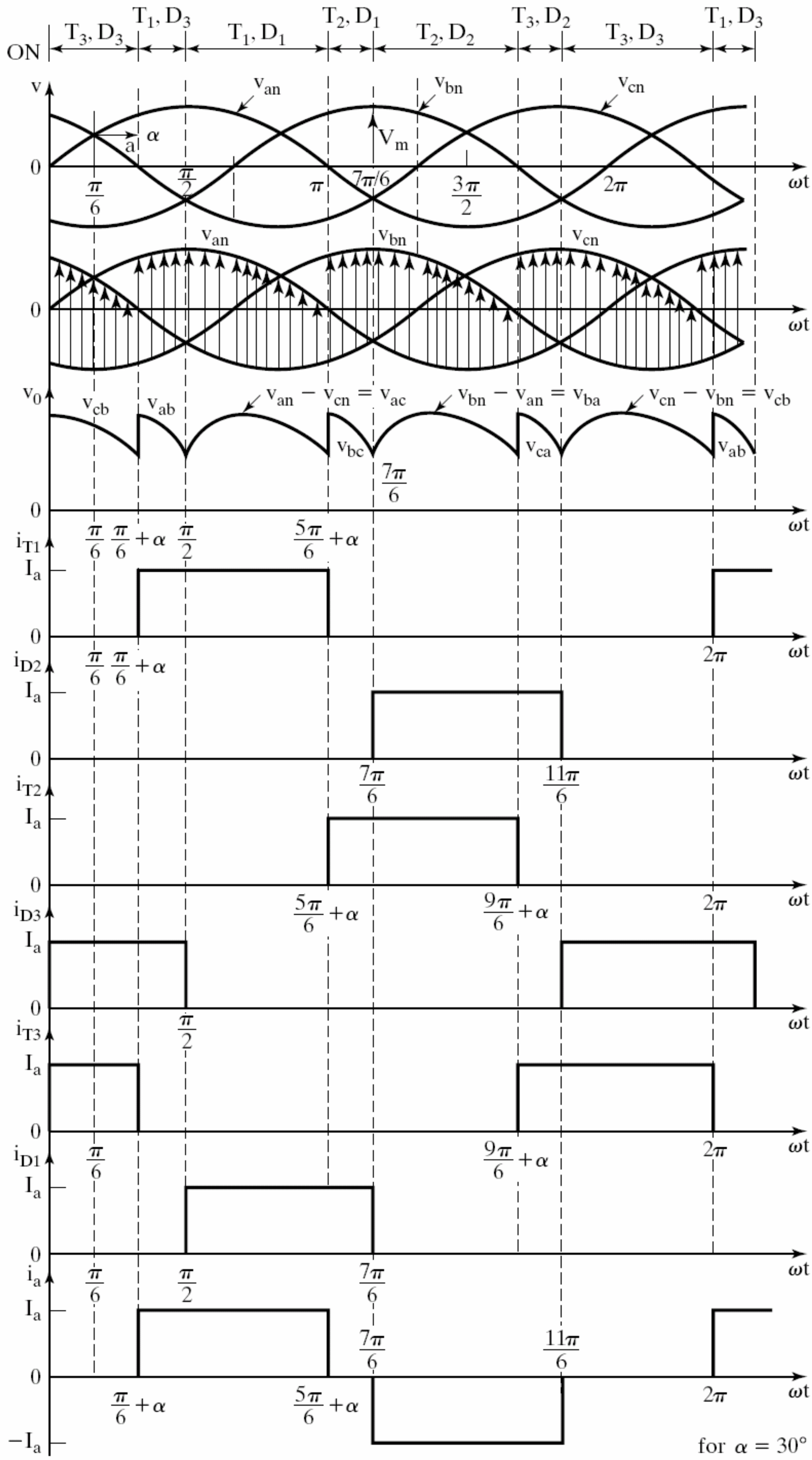
During the time period $\left(\frac{\pi}{6}\right) \le \omega t \le \left(\frac{7\pi}{6}\right)$ i.e., for $30^0 \le \omega t \le 210^0$, thyristor T_1 is forward biased. If T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right)$, T_1 and D_1 conduct together and the line to line supply voltage v_{ac} appears across the load. At $\omega t = \left(\frac{7\pi}{6}\right)$, v_{ac} starts to become negative and the free wheeling diode D_m turns on and conducts. The load current continues to flow through the free wheeling diode D_m and thyristor T_1 and diode D_1 are turned off.



(b) Waveforms for $\alpha = 90°$

If the free wheeling diode D_m is not connected across the load, then T_1 would continue to conduct until the thyristor T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha\right)$ and the free wheeling action is accomplished through T_1 and D_2, when D_2 turns on as soon as v_{an}

becomes more negative at $\omega t = \left(\frac{7\pi}{6}\right)$. If the trigger angle $\alpha \le \left(\frac{\pi}{3}\right)$ each thyristor conducts for $\frac{2\pi}{3}$ radians $\left(120^0\right)$ and the free wheeling diode D_m does not conduct. The waveforms for a 3-phase semi-converter with $\alpha \le \left(\frac{\pi}{3}\right)$ is shown in figure



ON
T3, D3
T1, D3
T1, D1
T2, D1
T2, D2
T3, D2
T3, D3
T1, D3
v
van
vbn
vcn
a
α
Vm
0
π/6
π/2
π
7π/6
3π/2
2π
ωt
v0
vcb
vab
van − vcn = vac
vbn − van = vba
vcn − vbn = vcb
vbc
vca
vab
7π/6
iT1
Ia
π/6
π/6 + α
π/2
5π/6 + α
iD2
7π/6
11π/6
iT2
5π/6 + α
9π/6 + α
iD3
iT3
iD1
ia
−Ia
for α = 30°

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin\omega t; \quad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t - 240^0\right)$$

The corresponding line-to-line voltages are

$$v_{RB} = v_{ac} = \left(v_{an} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)$$

$$v_{YR} = v_{ba} = \left(v_{bn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{5\pi}{6}\right)$$

$$v_{BY} = v_{cb} = \left(v_{cn} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

Where V_m is the peak phase voltage of a star (Y) connected source.

TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF THREE PHASE SEMICONVERTER FOR $\alpha > \left(\frac{\pi}{3}\right)$ AND DISCONTINUOUS OUTPUT VOLTAGE

For $\alpha \geq \frac{\pi}{3}$ and discontinuous output voltage: the average output voltage is found from

$$V_{dc} = \frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} v_{ac}.d\left(\omega t\right)$$

$$V_{dc} = \frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} \sqrt{3}\ V_m \sin\left(\omega t - \frac{\pi}{6}\right) d(\omega t)$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$$

$$V_{dc} = \frac{3V_{mL}}{2\pi}(1+\cos\alpha)$$

The maximum average output voltage that occurs at a delay angle of $\alpha = 0$ is

$$V_{dm} = \frac{3\sqrt{3}V_m}{\pi}$$

The normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1+\cos\alpha)$$

The rms output voltage is found from

$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} 3V_m^2 \sin^2\left(\omega t - \frac{\pi}{6}\right) d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \sqrt{3}V_m \left[\frac{3}{4\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

For $\alpha \le \frac{\pi}{3}$, and continuous output voltage

Output voltage $v_O = v_{ab} = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$; for $\omega t = \left(\frac{\pi}{6} + \alpha\right)$ to $\left(\frac{\pi}{2}\right)$

Output voltage $v_O = v_{ac} = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)$; for $\omega t = \left(\frac{\pi}{2}\right)$ to $\left(\frac{5\pi}{6} + \alpha\right)$

The average or dc output voltage is calculated by using the equation

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{\pi/2} v_{ab}.d(\omega t) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}.d(\omega t)\right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$$

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1+\cos\alpha)$$

The RMS value of the output voltage is calculated by using the equation

$$V_{O(RMS)} = \left[\frac{3}{2\pi}\int_{\pi/6+\alpha}^{\pi/2} v_{ab}^2.d(\omega t) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}^2.d(\omega t)\right]^{\frac{1}{2}}$$

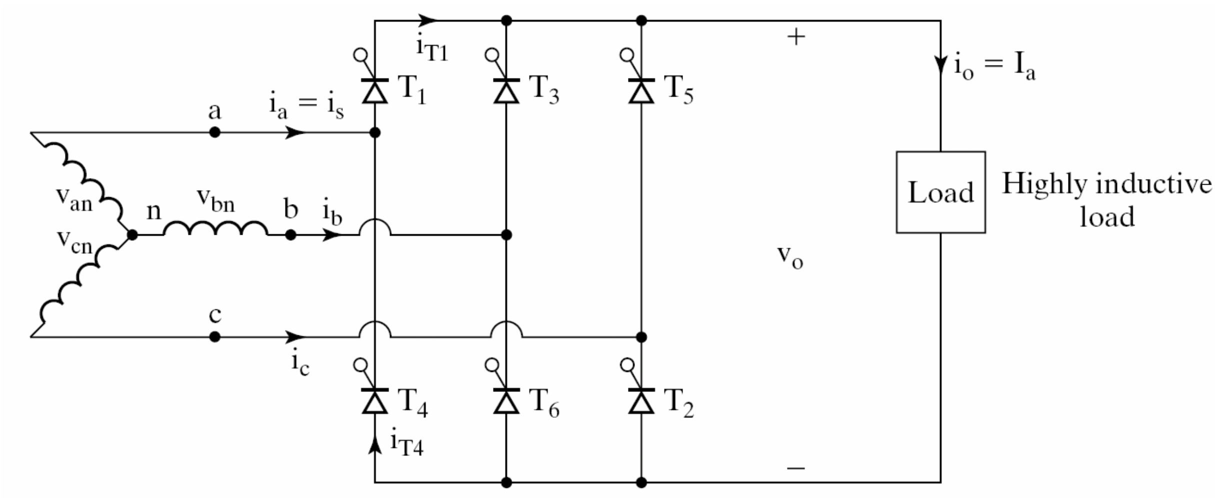
$$V_{O(RMS)} = \sqrt{3}V_m\left[\frac{3}{4\pi}\left(\frac{2\pi}{3}+\sqrt{3}\cos^2\alpha\right)\right]^{\frac{1}{2}}$$

THREE PHASE FULL CONVERTER

Three phase full converter is a fully controlled bridge controlled rectifier using six thyristors connected in the form of a full wave bridge configuration. All the six thyristors are controlled switches which are turned on at a appropriate times by applying suitable gate trigger signals.

The three phase full converter is extensively used in industrial power applications upto about 120kW output power level, where two quadrant operation is required. The figure shows a three phase full converter with highly inductive load. This circuit is also known as three phase full wave bridge or as a six pulse converter.

The thyristors are triggered at an interval of $\left(\frac{\pi}{3}\right)$ radians (i.e. at an interval of 60^0). The frequency of output ripple voltage is $6f_S$ and the filtering requirement is less than that of three phase semi and half wave converters.



At $\omega t = \left(\frac{\pi}{6}+\alpha\right)$, thyristor T_6 is already conducting when the thyristor T_1 is turned on by applying the gating signal to the gate of T_1. During the time period $\omega t = \left(\frac{\pi}{6}+\alpha\right)$ to $\left(\frac{\pi}{2}+\alpha\right)$, thyristors T_1 and T_6 conduct together and the line to line supply voltage v_{ab} appears across the load.

At $\omega t = \left(\frac{\pi}{2}+\alpha\right)$, the thyristor T_2 is triggered and T_6 is reverse biased immediately and T_6 turns off due to natural commutation. During the time period $\omega t = \left(\frac{\pi}{2}+\alpha\right)$ to $\left(\frac{5\pi}{6}+\alpha\right)$, thyristor T_1 and T_2 conduct together and the line to line supply voltage v_{ac} appears across the load.

The thyristors are numbered in the circuit diagram corresponding to the order in which they are triggered. The trigger sequence (firing sequence) of the thyristors is 12, 23, 34, 45, 56, 61, 12, 23, and so on. The figure shows the waveforms of three phase input supply voltages, output voltage, the thyristor current through T_1 and T_4 , the supply current through the line 'a'.

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin\omega t \quad ; \qquad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right) == V_m \sin\left(\omega t - 240^0\right)$$

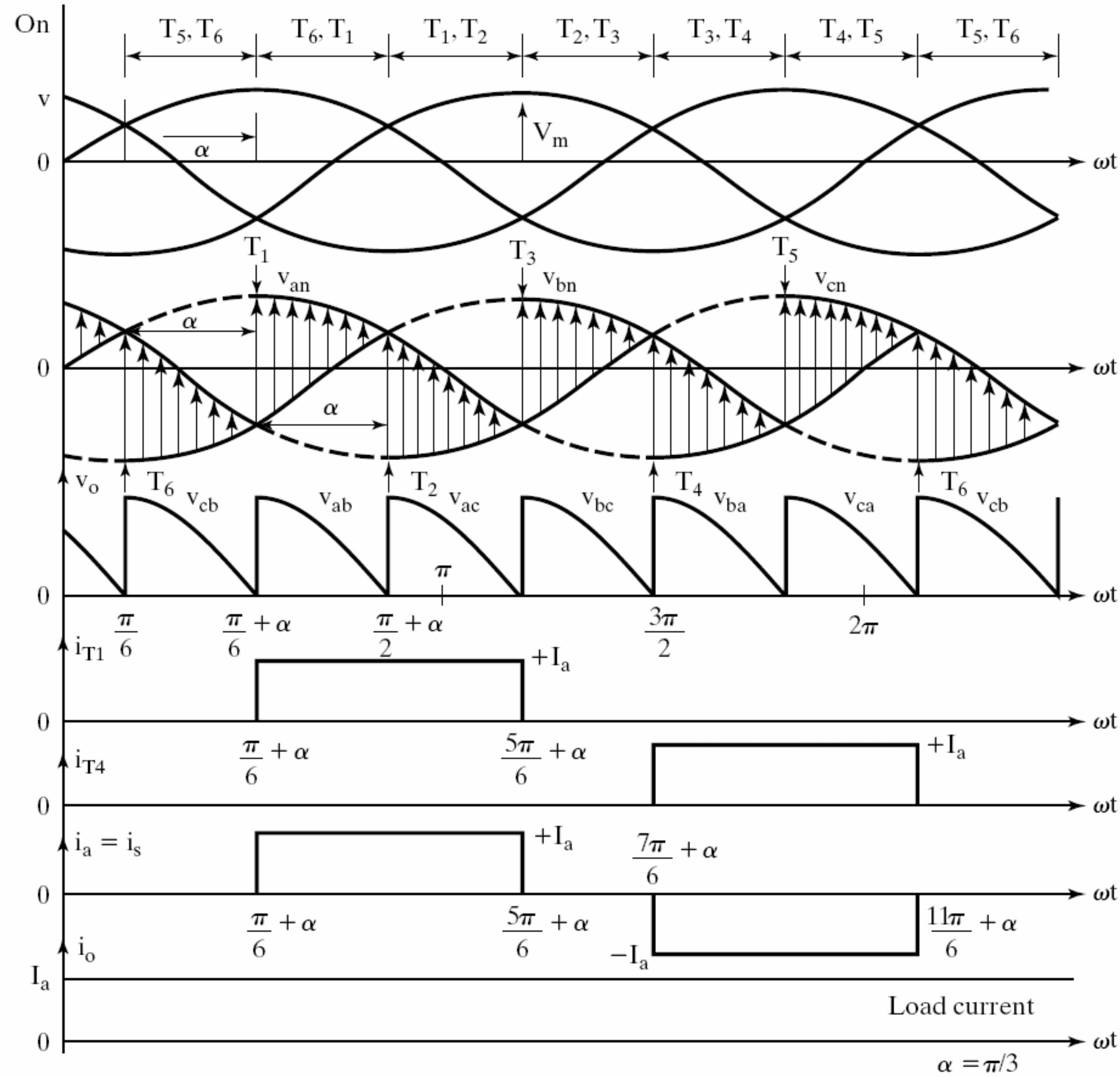
Where V_m is the peak phase voltage of a star (Y) connected source.

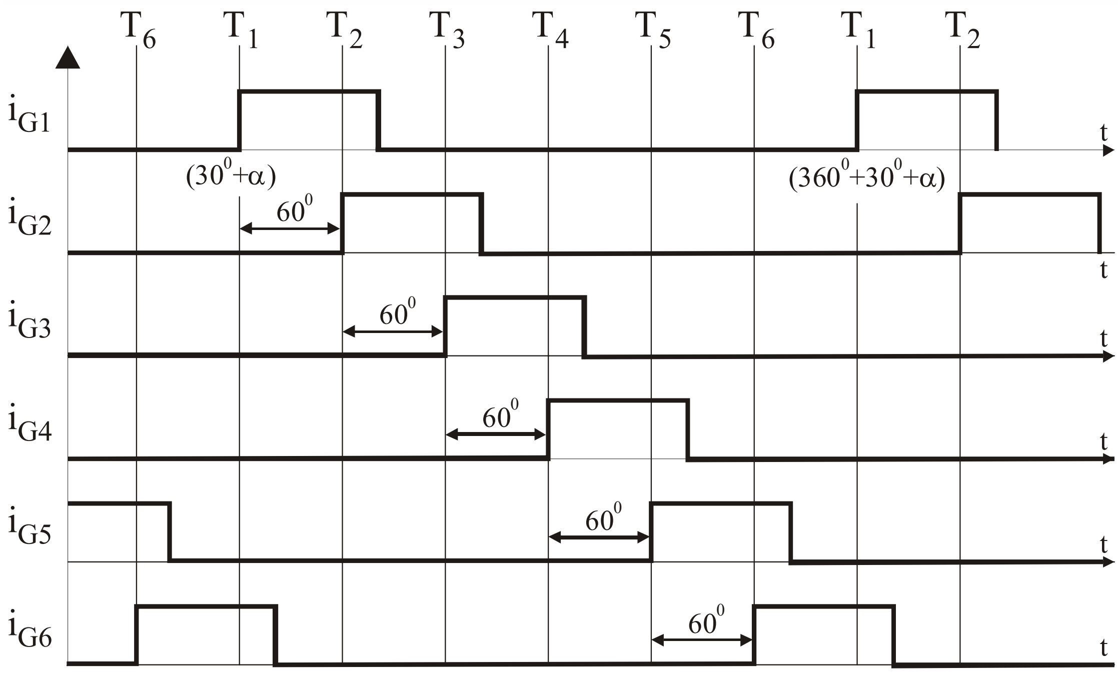
The corresponding line-to-line voltages are

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = \left(v_{bn} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = \left(v_{cn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$





Gating (Control) Signals of 3-phase full converter

TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF THREE PHASE FULL CONVERTER WITH HIGHLY INDUCTIVE LOAD ASSUMING CONTINUOUS AND CONSTANT LOAD CURRENT

The output load voltage consists of 6 voltage pulses over a period of 2π radians, hence the average output voltage is calculated as

$$V_{O(dc)} = V_{dc} = \frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O.d\omega t \qquad ;$$

$$v_O = v_{ab} = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{dc} = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right).d\omega t$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha = \frac{3V_{mL}}{\pi}\cos\alpha$$

Where $V_{mL} = \sqrt{3}V_m =$ Max. line-to-line supply voltage

The maximum average dc output voltage is obtained for a delay angle $\alpha = 0$,

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi} = \frac{3V_{mL}}{\pi}$$

The normalized average dc output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

The rms value of the output voltage is found from

$$V_{O(rms)} = \left[\frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O^2.d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_{ab}^2 . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

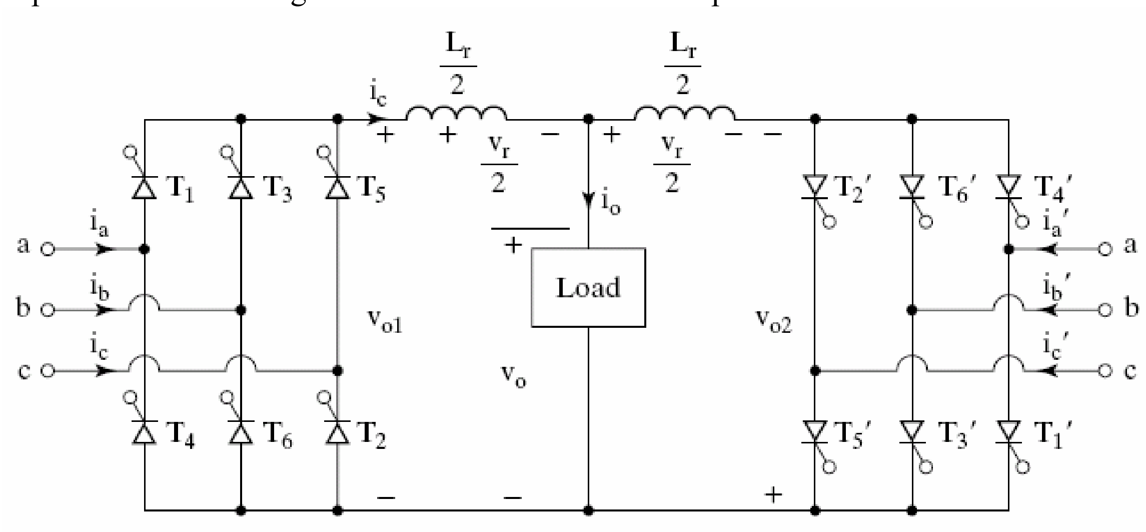
$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} 3V_m^2 \sin^2\left(\omega t + \frac{\pi}{6}\right) . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3} V_m \left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha \right)^{\frac{1}{2}}$$

THREE PHASE DUAL CONVERTERS

In many variable speed drives, the four quadrant operation is generally required and three phase dual converters are extensively used in applications up to the 2000 kW level. Figure shows three phase dual converters where two three phase full converters are connected back to back across a common load. We have seen that due to the instantaneous voltage differences between the output voltages of converters, a circulating current flows through the converters. The circulating current is normally limited by circulating reactor, L_r. The two converters are controlled in such a way that if α_1 is the delay angle of converter 1, the delay angle of converter 2 is $\alpha_2 = (\pi - \alpha_1)$.

The operation of a three phase dual converter is similar that of a single phase dual converter system. The main difference being that a three phase dual converter gives much higher dc output voltage and higher dc output power than a single phase dual converter system. But the drawback is that the three phase dual converter is more expensive and the design of control circuit is more complex.



The figure below shows the waveforms for the input supply voltages, output voltages of converter1 and conveter2 , and the voltage across current limiting reactor (inductor) L_r. The operation of each converter is identical to that of a three phase full converter.

During the interval $\left(\frac{\pi}{6} + \alpha_1\right)$ to $\left(\frac{\pi}{2} + \alpha_1\right)$, the line to line voltage v_{ab} appears across the output of converter 1 and v_{bc} appears across the output of converter 2

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \quad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

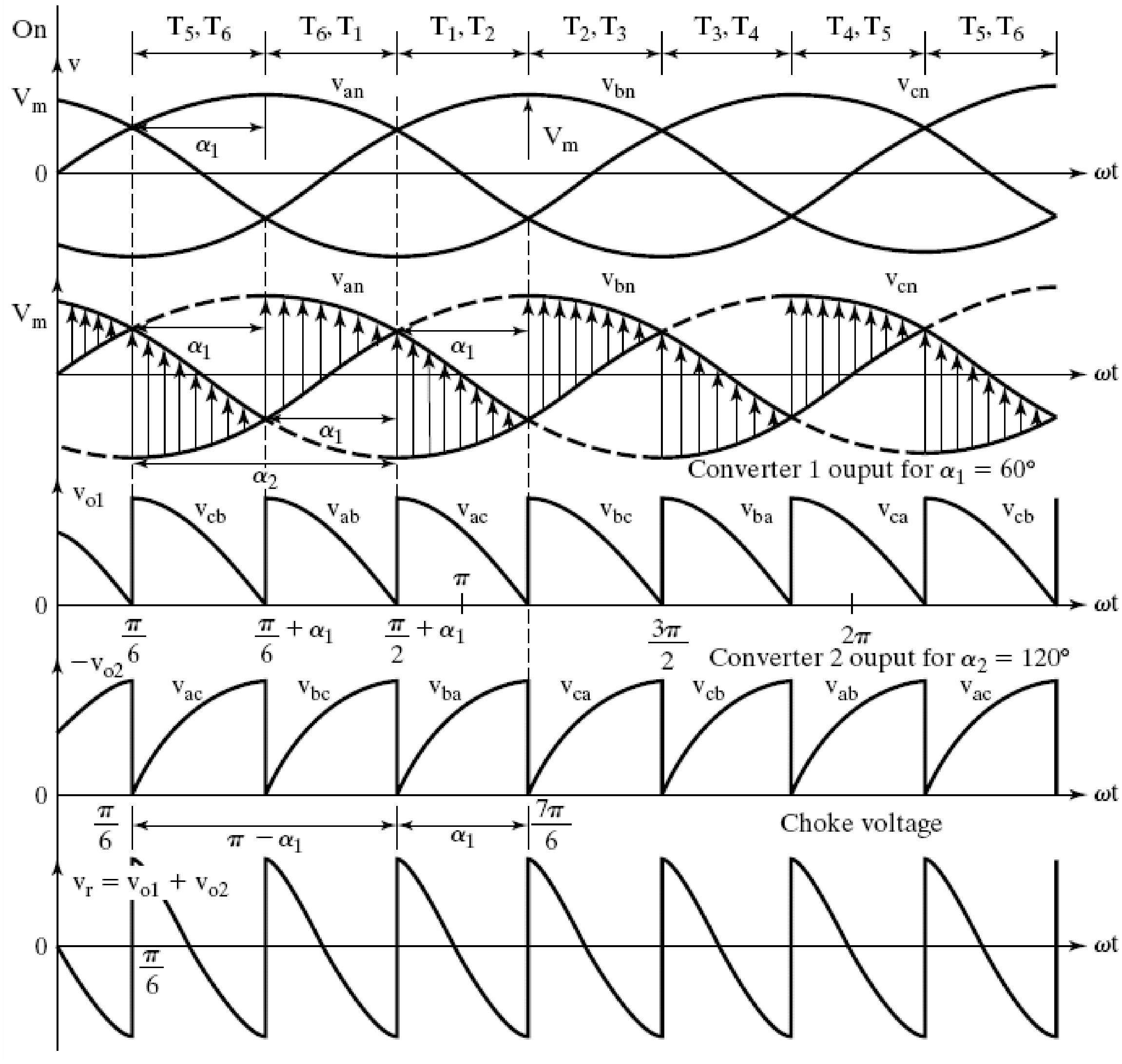
$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right) = V_m \sin\left(\omega t - 240^0\right)$$

The corresponding line-to-line supply voltages are

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = \left(v_{bn} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = \left(v_{cn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$



TO OBTAIN AN EXPRESSION FOR THE CIRCULATING CURRENT

If v_{O1} and v_{O2} are the output voltages of converters 1 and 2 respectively, the instantaneous voltage across the current limiting inductor during the interval $\left(\frac{\pi}{6}+\alpha_1\right) \le \omega t \le \left(\frac{\pi}{2}+\alpha_1\right)$ is

$$v_r = \left(v_{O1} + v_{O2}\right) = \left(v_{ab} - v_{bc}\right)$$

$$v_r = \sqrt{3}V_m \left[\sin\left(\omega t + \frac{\pi}{6}\right) - \sin\left(\omega t - \frac{\pi}{2}\right) \right]$$

$$v_r = 3V_m \cos\left(\omega t - \frac{\pi}{6}\right)$$

The circulating current can be calculated by using the equation

$$i_r\left(t\right) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6}+\alpha_1}^{\omega t} v_r . d\left(\omega t\right)$$

$$i_r\left(t\right) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6}+\alpha_1}^{\omega t} 3V_m \cos\left(\omega t - \frac{\pi}{6}\right) . d\left(\omega t\right)$$

$$i_r\left(t\right) = \frac{3V_m}{\omega L_r} \left[\sin\left(\omega t - \frac{\pi}{6}\right) - \sin\alpha_1 \right]$$

$$i_{r(\max)} = \frac{3V_m}{\omega L_r} \quad = \text{ maximum value of the circulating current.}$$

There are two different modes of operation of a three phase dual converter system.

- Circulating current free (non circulating) mode of operation
- Circulating current mode of operation

CIRCULATING CURRENT FREE (NON-CIRCULATING) MODE OF OPERATION

In this mode of operation only one converter is switched on at a time when the converter number 1 is switched on and the gate signals are applied to the thyristors the average output voltage and the average load current are controlled by adjusting the trigger angle α_1 and the gating signals of converter 1 thyristors.

The load current flows in the downward direction giving a positive average load current when the converter 1 is switched on. For $\alpha_1 < 90^0$ the converter 1 operates in the rectification mode V_{dc} is positive, I_{dc} is positive and hence the average load power P_{dc} is positive.

The converter 1 converts the input ac supply and feeds a dc power to the load. Power flows from the ac supply to the load during the rectification mode. When the trigger angle α_1 is increased above 90^0, V_{dc} becomes negative where as I_{dc} is positive because the thyristors of converter 1 conduct in only one direction and reversal of load current through thyristors of converter 1 is not possible.

For $\alpha_1 > 90^0$ converter 1 operates in the inversion mode & the load energy is supplied back to the ac supply. The thyristors are switched-off when the load current decreases to zero & after a short delay time of about 10 to 20 milliseconds, the converter 2 can be switched on by releasing the gate control signals to the thyristors of converter 2.

We obtain a reverse or negative load current when the converter 2 is switched ON. The average or dc output voltage and the average load current are controlled by adjusting the trigger angle α_2 of the gate trigger pulses supplied to the thyristors of converter 2. When α_2 is less than 90^0, converter 2 operates in the rectification mode and converts the input ac supply in to dc output power which is fed to the load.

When α_2 is less than 90^0 for converter 2, V_{dc} is negative & I_{dc} is negative, converter 2 operates as a controlled rectifier & power flows from the ac source to the load circuit. When α_2 is increased above 90^0, the converter 2 operates in the inversion mode with V_{dc} positive and I_{dc} negative and hence P_{dc} is negative, which means that power flows from the load circuit to the input ac supply.

The power flow from the load circuit to the input ac source is possible if the load circuit has a dc source of appropriate polarity.

When the load current falls to zero the thyristors of converter 2 turn-off and the converter 2 can be turned off.

CIRCULATING CURRENT MODE OF OPERATION

Both the converters are switched on at the same time in the mode of operation. One converter operates in the rectification mode while the other operates in the inversion mode. Trigger angles α_1 & α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^0$

When $\alpha_1 < 90^0$, converter 1 operates as a controlled rectifier. When α_2 is made greater than 90^0, converter 2 operates in the inversion mode. V_{dc}, I_{dc}, P_{dc} are positive.

When $\alpha_2 < 90^0$, converter 2 operates as a controlled rectifier. When α_1 is made greater than 90^0, converter 1 operates as an Inverter. V_{dc} and I_{dc} are negative while P_{dc} is positive.

Problems

1. A 3 phase fully controlled bridge rectifier is operating from a 400 V, 50 Hz supply. The thyristors are fired at $\alpha = \frac{\pi}{4}$. There is a FWD across the load. Find the average output voltage for $\alpha = 45^0$ and $\alpha = 75^0$.

Solution

For $\alpha = 45^0$, $V_{dc} = \frac{3V_m}{\pi}\cos\alpha$

$$V_{dc} = \frac{3 \times \sqrt{2} \times 400}{\pi}\cos 45^0 = 382 \text{ Volts}$$

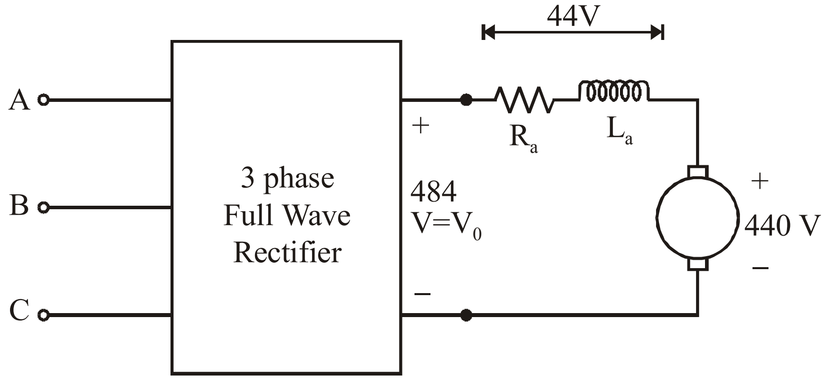
For $\alpha = 75^0$, $V_{dc} = \frac{6V_m}{2\pi}\left[1+\cos\left(60^0+\alpha\right)\right]$

$$V_{dc} = \frac{6 \times \sqrt{2} \times 400}{2\pi}\left[1+\cos\left(60^0+75^0\right)\right]$$

$$V_{dc} = 158.4 \text{ Volts}$$

2. A 6 pulse converter connected to 415 V ac supply is controlling a 440 V dc motor. Find the angle at which the converter must be triggered so that the voltage drop in the circuit is 10% of the motor rated voltage.

Solution



R_a - Armature resistance of motor.

L_a - Armature Inductance.

If the voltage across the armature has to be the rated voltage i.e., 440 V, then the output voltage of the rectifier should be 440 + drop in the motor

That is $\quad 440 + 01 \times 440 = 484 \text{ Volts}$.

Therefore $V_O = \frac{3V_m \cos\alpha}{\pi} = 484$

That is $\frac{3 \times \sqrt{2} \times 415 \times \cos\alpha}{\pi} = 484$

Therefore $\alpha = 30.27^0$

3. A 3 phase half controlled bridge rectifier is feeding a RL load. If input voltage is 400 sin314t and SCR is fired at $\alpha = \frac{\pi}{4}$. Find average load voltage. If any one supply line is disconnected what is the average load voltage.

Solution

$\alpha = \frac{\pi}{4}$ radians which is less than $\frac{\pi}{3}$

Therefore $V_{dc} = \frac{3V_m}{2\pi}\left[1 + \cos\alpha\right]$

$$V_{dc} = \frac{3 \times 400}{2\pi}\left[1 + \cos 45^0\right]$$

$$V_{dc} = 326.18 \text{ Volts}$$

If any one supply line is disconnected, the circuit behaves like a single phase half controlled rectifies with RL load.

$$V_{dc} = \frac{V_m}{\pi}\left[1 + \cos\alpha\right]$$

$$V_{dc} = \frac{400}{\pi}\left[1 + \cos 45^0\right]$$

$$V_{dc} = 217.45 \text{ Volts}$$